ARITHMETIC ALGEBRA

PART I

MATHEMATICS FOR MIDDLE SCHOOLS

INSTITUTE OF EDUCATION FOR WOMEN ★ CALCUTTA-1
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FOREWORD

In the second half of the twentieth century, the importance of science and technology to every nation does not need to be argued. It is evident to the most casusal observer. The explosive expansion of fundamental scientific knowledge that began in the first half of the century is a potential source of enduring improvement in a nation's economy, capable of bringing to every citizen more food and water, better clothing and shelter, greater opportunities for education and wage-earning and, in general, a healthier, safer and more rewarding life. But if a nation is to be able to exploit modern science and technology, it must have men and women competent to do fundamental scientific research and men and women competent to translate the resulting knowledge into means for improvement of the national economy.

What is true of all nations is true in a special degree of developing societies such as India's. If the recognition of one truth in education in the twentieth century is more valuable than any other, it is that education is the most potent instrument for change anywhere. India, the developing society *par excellence*, is at the stage of having to pull herself up by the bootstraps out of the rut of a traditional society to move swiftly and effectively into the orbit of modernity that is a precondition of national survival.

Improvement in the total standard of science and mathematics is a continuous process that demands three integrated steps, and the process begins necessarily at school level. It moves as follows :

(a) Development of a curriculum that includes modern concepts and understandings of the subject-fields and a rigorous, analytic study of fundamentals.

(b) Preparation of textbooks based on the new curriculum ; teachers' guides and manuals and other instructional materials ; development of experimental kits and apparatus.

(c) Training teachers to enable them to introduce the new curriculum into the classroom.

Science in some form or other, either as general science or as everyday science is being taught in all our middle schools. It has, however, been realized by all scientists and educationists that the teaching of general science at the middle school stage is inadequate since it does not provide a firm foundation for the understanding of the basic concepts of physics, chemistry and biology.

The Education Commission (1964-66) has observed that "the general science approach to the teaching of science which has been widely adopted at the elementary stage during the last 10 years has not proved successful as it tends to make science appear somewhat formless and without structure and runs counter to its methodology. A disciplined approach to science learning would, it is felt, be more effective in providing the necessary scientific base to the young pupil".

A detailed survey of the present position of science and mathematics in our schools was also carried out by the National Council of Educational Research and Training. It revealed serious shortcomings. The curriculum is for the most part dated. It does not include modern concepts on the understanding of science. In mathematics the emphasis is on developing skills and not on a disciplined understanding of the basic fundamentals of the subject. No organic relationship exists in the teaching of the biological sciences, physical sciences and mathematics ; each falls into a separate pigeonhole. The textbooks used are of poor quality. Teachers have not been provided with manuals, guides and other instructional materials. Laboratory apparatus and equipment are still of the conventional type and unsuited to teaching science along modern lines.

A team of Soviet science educators visited India about three years ago and put in a report on the lines on which science and mathematics education should be improved in India's secondary schools. They recommended that instead of teaching general science, the individual disciplines of science structured as physics, chemistry and biology should be taught from the very beginning of the middle school stage.

Following upon these surveys and reports, the Department of Science Education in the National Council of Educational Research and Training has undertaken a major project to upgrade science and mathematics, starting from the middle school stage. Under this project, a new curriculum is being formulated, textbooks and teachers' guides prepared and teachers trained. For this purpose, a number of schools have been selected in Delhi where the new curriculum and other instructional materials developed by the Department, are being tried out in actual situations. Science is being taught not as general science but as physics, chemistry, biology and mathematics, that is, as separate subjects and in depth. On the basis of the experience gained, the curriculum and instructional materials are being refined and further developed for application throughout the country. The materials produced so far cover the three years of the middle school stage and have been subjected to rigorous trial and experiment in Delhi schools.

The present series of text materials now published consists of teaching units, textbooks, curriculum guides and teachers' manuals for the first year of the middle school stage. These have successfully passed through exhaustive try-outs and tests in selected schools and are now available for application throughout the country. The syllabi and text materials are based on an approach that advocates the teaching of the individual disciplines of science separately. The curriculum has a practical bias. The knowledge of science is given through experimental activities to be performed by teachers and pupils. Every care has been taken to present the correct and up to date facts of science and to see that the examples used are drawn from Indian situations. A variety of indigenous equipment and apparatus have been designed and produced by the Central Science Workshop of the National Council to make available better instructional materials to teachers.

The National Council is grateful to the Unesco experts who assisted the Department of Science Education on this project. We are also thankful to the Director of Education, Delhi Administration who very kindly provided facilities to try-out the curricular materials in 30 selected schools in Delhi.

The National Council welcomes the constructive comments and suggestions of teachers and others interested in science education for further improvement of the curricular materials.

L. S. CHANDRAKANT

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CHAPTER I

NATURAL NUMBERS

1. Decimal System

NUMBER AND NUMERAL

We get an idea of counting numbers, whenever we wish to know 'how many' in a collection of objects. For example, the set of objects in fig. 1.1 gives the idea of number 4.

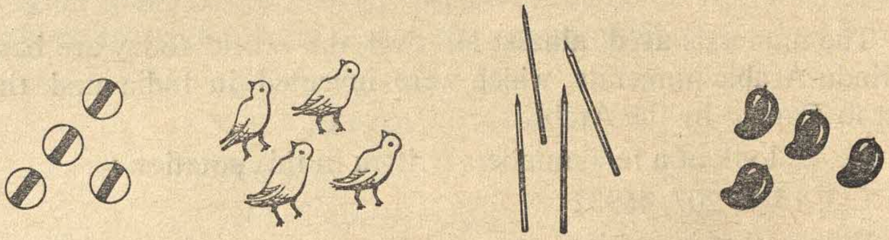

Fig. 1.1

In ancient times Babylonians, Egyptians, Romans and the men living in caves represented the idea of number 4 in the following ways. The Hindu-Arabic notation is also shown here.

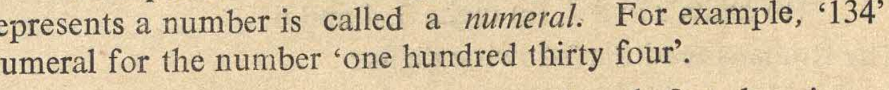

Fig. 1.2

We represent a number by using symbols. The symbol which represents a number is called a *numeral.* For example, '134' is the numeral for the number 'one hundred thirty four'.

The number ideas were communicated first by pictures and finally by numerical symbols. The cave-men represented the numbers by pictures, strokes, notches, tally-sticks and knots in strings and later by putting pebbles in a pile.

The Egyptians denoted the numbers by pictures of ropes, flowers, bones etc., as shown in fig. 1.3.

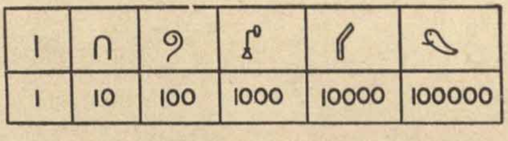
1
10
100
1000
10000
100000

Fig. 1.3

The Babylonians used the symbols shown in fig. 1.4.

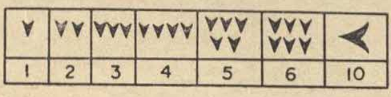

Fig. 1.4

The numerals used almost all over the world today are based on Hindu-Arabic numerals which were invented in India and then taken to Europe by the Arabs.

Let us look at a few numbers written in this notation :

123, 1538, 207, 46932

We see that in writing any number we use only the following ten symbols :

1,2,3,4,5,6,7,8,9 and 0

Each of these ten symbols is called a *digit.*

Numerals such as 8, 48, and 113 are made up of one or more digits. We refer to '8' as a one-digit numeral, to '48' as a two-digit numeral and to '113' as a three-digit numeral.

1. How many different digits are there in each of the following numerals ?

735, 486, 237, 00067, 5353, 99999

2. Write down three-digit numerals using (a) digit 2 (b) digits 2 and 4, (c) digits 3, 4 and 6

The Romans used the following digits :

I	V	X	L	C	M
one	five	ten	fifty	hundred	thousand

In this case while writing the numerals, the digits are usually placed in decreasing order. The digits, I, X, C, M may be repeated twice or three times. Thus in the digit XXXVII the digit X has been repeated three times and the digit I has been repeated twice.

To read the number expressed by the numeral in which the smaller digit always comes to the right of the greater digit, we add the values of all the digits.

Ex. 1 $\mathrm{XXXVII}=10+10+10+5+1+1=37$

3. What number is represented by :

   (a) VI (b) XVIII (c) CVII

This rule cannot be applied to such numbers, for example, IV, XIX, XL, XC, because in these cases the smaller digit is placed before the greater digit. In such cases the value of the smaller digit should be subtracted from that of the greater digit.

Ex. 2

$$\mathrm{IV}=5-1=4$$
$$\mathrm{XIX}=10+(10-1)=19$$
$$\mathrm{XL}=50-10=40$$
$$\mathrm{XC}=100-10=90$$

Ex. 3 $\mathrm{XXXIV}=10+10+10+(5-1)=34$

4. Write down the Hindu-Arabic numerals corresponding to: XII, XVIII, XXI, XXXVII IX, XIX
5. Write down the Roman numerals corresponding to : 13, 16, 17, 21, 28, 4, 9, 29, 36, 41
6. What numbers can you write using Roman digits I and V ?
7. What is the difference between the Roman numeral III and the international numeral 111 ?

Each collection of objects is indicated by a number. This number is found by counting the objects, *i.e.*, by associating a number to the objects in the collection.

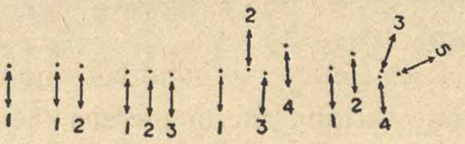

Fig. 1.5

8. What is the mistake in fig. 1.6 in counting the stars (Sapta Rishi) ?

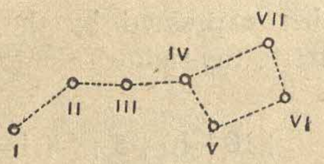

Fig. 1.6

The numbers that we use for counting are known as *natural numbers.*

If we write down these natural numbers in the order in which they are used in the process of counting, we get the row of natural numbers.

9. Is there an end to the row of natural numbers ? Is each number followed by a successor ?
10. Which number in the row of natural numbers follows the number 7 ; the number 14 ; the number 189 ; and the number 8645 ?
11. Write down all the possible three-digit numerals of the numbers that can be formed out of the digits 3, 5, 7 without repeating the same digit in the same numeral.

Base

Our system of numeration uses the following ten digits :

1, 2, 3, 4, 5, 6, 7, 8, 9, and 0

In writing the numeral for the number ten, we put 1 to the left of 0. Thus 10 is the numeral for the number ten. Here we used the same symbol which we used for one but we wrote the symbol in a different place which we call ten's place.

In writing eleven, the zero in 10 is replaced by 1. Thus eleven, one more than 10, is written as 11. We can go on replacing the zero in 10 by 2, 3, 4,.........9, to write the successive numerals. When we reach twenty we write it as 20. The process can be continued further as :

21, 22,......................., 29, 30 ;
31, 32......................., 39, 40

Thus it is clear that we group numbers in groups of ten. As the size of each basic group is 10, we say that the base of our system of numeration is ten.

Our system of numeration is called the 'Decimal System'. The word decimal means ten. Our system is called the decimal system because we use 10 symbols (digits) for writing all the numbers in our system.

11 is 1 ten and 1 one

75 is 7 tens and 5 ones

89 is 8 tens and 9 ones

99 is 9 tens and 9 ones

If we count one more than 99, we get ten tens or one hundred and we write it as 100. Here we use the symbol 1 and 0, but we write the symbol 1 in a different place. 2 places to the left of unit's place. This place is called the hundred's place.

Hundred's place	Ten's place	Unit's place	
	—	1	=one
	1	0	=one ten and no ones
	1	2	=one ten and two ones
1	0	0	=one hundred, no tens, no ones =100
4	3	8	=four hundreds, three tens and eight ones =438

In reading the numbers, we have to read the numeral with the name of each digit and the place it holds, beginning from the left.

It is clear that each place represents ten times the one which is immediately to its right. The first place (unit's place) tells us how many ones are there, the second place (ten's place) tells us how many groups of ten ones, *i.e.*, 10 are there, the third place (hundred's place) tells us how many groups of ten tens, *i.e.*, 100 are there and so on. It should be noted, therefore, that the number represented by a numeral depends on the position of the digits in the numeral.

12. What values are represented by fives in the following numerals ?
    215, 5001, 2550
13. Write down any four numbers using the digits 4, 2, 8, 6. Do not repeat the digits.
14. Using the digits 3, 8 and 9, write down :
    (a) three one-digit numbers
    (b) four two-digit numbers
15. Write down a seven-digit number which will not change if the digits are written in the reverse order, and read it.
16. In the numeral 5426027, what place value does each of the digits represent ?

2. Comparison of Numbers

Comparison of two numbers means finding out which of them is smaller or greater than the other.

Out of two numbers, one is considered to be less than the other if it is nearer to the beginning of the row of natural numbers than the other. A number is considered to be greater than the other if it is farther than the other from the beginning of the row of natural numbers. So the number 53 is less than the number 75, the number 18 is greater than the number 15.

Instead of using the words 'less than' and 'greater than', the signs '<' and '>' are used to denote them respectively. So the sentence 'fifty nine is less than seventy five' can be written in a shorter way as $59<75$ and the sentence 'eighteen is greater than fifteen,' can be written as $18>15$. These sentences are called **'Inequalities'** and the signs '<' and '>' are called the signs of inequality.

17. Read the following inequalities :
    $608<726$; $32447>15$
18. Compare the following pairs of numbers and put the correct signs of inequality in place of the star :
    (a) 68708 * 69006
    (b) 80008 * 78888

19. What is the smallest natural number ?

20. Compare the following numbers :

(a) 300000 and 3000 (b) 5555 and 0

Now we shall illustrate how to represent natural numbers on a line.

Draw a line as shown in fig. 1.7. Select a point on it and denote it by 0. Mark points to the right of 0 at equal intervals. Denote the first point to the right of 0 by 1 and the next point to the right of 1 by the next counting number 2. Similarly, assign the numbers 3, 4, 5,......to the other points on the line. Now we get a picture of the natural numbers. This picture represents the *number-line.*

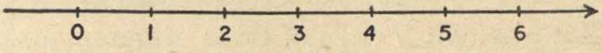

Fig. 1.7

Note that each number except zero has a natural number to its right and a natural number to its left. Also note that each number is greater than the number to its left by one and it is less than the number to its right by one. For example, 6 is greater than 5 by one and 6 is less than 7 by one. Similarly, 5 is greater than 4 by one and 7 is less than 8 by one. Since 5 is greater than 4 and 6 is greater than 5, we say that 6 is greater than 4. Reasoning in the same way we say that 6 is greater than all the numbers to its left. Similarly, we can say that 6 is less than all the numbers to its right.

If the sign of inequality is put in the wrong way, we say that the inequality is *false*. If it is put in the right way we say that the inequality is *true*. For example, $3<6$ is a true inequality whereas $15>31$ is a false inequality.

21. State whether the following inequalities are true or false :

(a) $3685>41500$ (c) $3651<1261$

(b) $2856>3264$ (d) $1782<3686$

Again, look at the number-line (*fig. 1.7*)., As we move to the right of 1 we find that each number is greater than its preceding number. We say that the numbers are arranged in an increasing order from left to right. If we move from right to left, we find that the numbers are arranged in a decreasing order.

22. Rearrange the following numbers in the increasing order : 874, 14, 606, 10001, 1
Express the ideas by using the sign of inequality ‘$<$’.

23. Rearrange the following numbers in the decreasing order and express the idea in the decreasing order, using the sign of inequality ‘$>$’ : 210, 45, 400, 5, 86

24. Draw a number-line and mark on it the points corresponding to numbers 1, 6, 9 and 10.
Also mark on the same line the numbers left over between 1 and 10.

25. There is a number which ends with the digit 5 and is more than 210 and less than 235. What is that number ? How many such numbers can you find ?

26. Ram bought 7 balls of three different colours, blue, red and green. The number of red balls is greater than the number of green balls and the number of blue balls is less than the number of green balls. How many balls of each colour are there ?

27. Write down all the possible inequalities using two of the numbers at a time : 202, 380, 445

28. How do you place 8 men into three cars A, B, and C such that there are more people in car C than in car B and there are more people in car B than in car A ?

3. Addition

We have read about addition in earlier classes. Take any two natural numbers, say, 5 and 7. If we put these together or combine them we get another natural number 12. We write $5+7=12$.

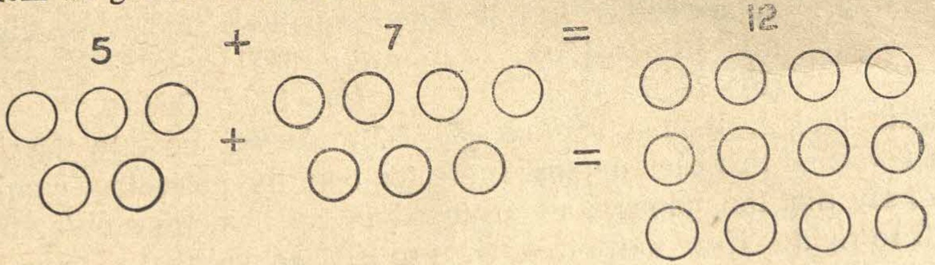

Fig. 1.8

The numbers 5 and 7 which we added are called the *addends* and the number 12 which we obtained as a result of addition is called the *sum*. Any two natural numbers can be added and their sum can be obtained. For example :

$$8+9=17,\ 25+7=32,\ 24+12=36,\ 532+124=656$$

Are all the numbers, that we get as a sum, natural numbers ? Can you think of any two natural numbers whose sum is not a natural number ? Can you find any number other than 17 which is the sum of 8 and 9 ? Is there more than one correct answer for the sum of any two natural numbers ?

We see that if we add two natural numbers, we get one and only one answer and the answer which we call the sum is another natural number.

29. What is the maximum number of digits possible in the sum if we add two 3-digit numbers ?
30. Is the following addition correct ?
    $4706+9885=14591$
31. Find the sum of :
    (a) the greatest 6-digit number and the least 3-digit number
    (b) the greatest 7-digit number and the least 3-digit number
32. Find the sums in the following :
    (a) $765+4651+235$
    (b) $2106+1192+909$

Commutative Law of Addition

The teacher read out two large numbers to be added. One boy did not hear the first number. He wrote the second number and then asked the teacher to repeat the first number.

The boy wrote : 2734 Others wrote : 6425
6425 2734
—— ——

—— ——

If every pupil added correctly, what can we say about the order of the addends and their sum in both the cases ?

From the above example we see that the order in which we add a pair of natural numbers does not affect their sum. This property is known as the *Commutative Law of Addition* for natural numbers. It can be stated thus : **If** a **and** b **represent any two natural numbers then** $a+b=b+a$.

In everyday life we speak of 'adding' or 'combining' things or actions. Whether such combinations have the commutative property or not depends upon the nature of the things or actions that we combine.

33. Consider the two operations : opening the window and putting the head through the window. Are these operations commutative ?
34. Consider the operation taking a right-turn and then taking an about-turn. Are these operations commutative ?
35. Give examples of non-commutative operations.
36. Give two examples of commutative operations.
37. Express each of the numbers 102 and 204 as the sum of two numbers, one of which is greater than the other by 2.

Associative Law of Addition

Suppose we are given three numbers 7, 9 and 11. We can find the sum of these numbers in any of the following ways :

(a) $7+9=16$ (b) $7+11=18$ (c) $9+11=20$
and $16+11=27$ and $18+9=27$ and $20+7=27$
$\therefore\ 7+9+11=27$ $\therefore\ 7+9+11=27$ $\therefore\ 7+9+11=27$

We add, if the number of addends is more than two, by grouping. For example, to find the sum of three numbers we first pair or group two numbers and find their sum ; then we pair this sum and the third number and find the sum. The pairing or grouping of the numbers is denoted by using the sign ( ) called a pair of brackets. Thus the example above can be expressed as follows :

(a) $(7+9)+11$ (b) $(7+11)+9$ (c) $7+(9+11)$

In all the above cases we are using a different grouping. Study the sum in each case. Do you find any difference in the sum ?

We notice that the order of grouping the numbers does not affect their sum. This is called the *Associative Law of Addition.*

38. Add 793 to the sum of 532 and 312. Check your answer by adding 312 to the sum of 532 and 793.
39. Simplify and point out which grouping makes the simplification easier :

    (a) $(4709+386)+3291$ (b) $4709+(386+3291)$

    (c) $(4709+3291)+386$
40. Find the sum of 29 and 7.

The associative law can be used in finding the sum of two numbers easily. Notice how we can use it ?

$$\begin{aligned}29+7&=(20+9)+7\\&=20+(9+7)\\&=20+16\\&=20+(10+6)\\&=(20+10)+6\\&=36\end{aligned}\qquad\begin{aligned}29+7&=(20+9)+(1+6)\\&=20+(9+1)+6\\&=20+10+6\\&=(20+10)+6\\&=30+6\\&=36\end{aligned}$$

The associative law of addition states that

if a, b and c represent any natural numbers then

$$(a+b)+c=a+(b+c)$$

In everyday life we speak of adding or combining several things. Whether such combinations have the associative property or not, depends on the nature of the things that we combine.

41. Is (gasoline+fire)+water the same as gasoline+(fire +water) ?
42. Give an example of three operations which are associative.

43. Give an example of three operations which are not associative.

44. Find the sum of :

(a) $381+119+207+293$

(b) $483+296+117+304$

45. Find the rule according to which the numbers are placed in fig. 1.9. Fill the numbers in the blanks according to this rule.

	16	13	12
	17	14	15
	18	19	20

Fig. 1.9

4. Subtraction

In earlier classes you have read subtraction as a process of 'taking away' or finding 'what is left'.

I had 12 pencils. I gave 4 pencils to Ram. How many pencils were left with me ?

We solve it as $12-4=8$. We say that if we subtract the natural number 4 (called the *subtrahend*) from the natural number 12 (called *minuend*) we get the natural number 8 (called the *difference*).

46. Simplify the following :

(a) $53251-4521$ (b) $11000-7649$

Which of the two numbers, the subtrahend or the minuend is greater in the above examples ?

47. Can you solve the following examples ?

(a) $3-5$ (b) $12-25$ (c) $89-356$

Can you find a natural number for the difference of two natural numbers, when the subtrahend is greater than the minuend ? We say that in the system of natural numbers, it is not always possible to perform the operation of subtraction.

Is subtraction commutative ? To find out an answer, check whether $a-b$ is equal to $b-a$ for any natural numbers a and b. What is your conclusion ?

Is there an associative property for subtraction ?

48. Is the subtraction done below, correct ?

    $8072-4968=3104$

    How do you call each of the numbers in the above equality ?

49. Find the difference between the smallest five-digit number and the greatest four-digit number.

50. There were 2084 quintals of potatoes in the store. How many quintals of potatoes were left, if 995 quintals were sold ?

51. Represent the number 2801 as the difference of two numbers, one of which ends with the digit 5.

52. Represent the number 779 as the difference of two numbers such that the minuend ends with the digit 2, or the digit 4.

53. Find the difference between the smallest ten-digit number and the greatest eight-digit number.

54. Represent the number 3012 as the sum of two numbers, one of which ends with the digit 9.

55. In fig. 1.10 are shown the paths along which two men, P and Q walk. P walks from the point A to the point C,

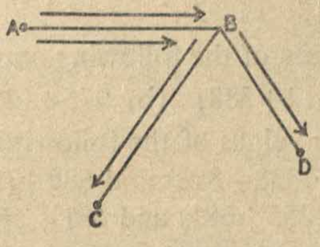

Fig. 1.10

and Q walks from the point A to the point D. Which distance is shorter ?

5. Expressions and Brackets

We have learnt how the operations of addition and subtraction are performed on two numbers. We have represented these operations symbolically, for example, $3+4$, $8-3$. We have also applied these operations to groups of three or more numbers, for example, $3+5+7$, $315+519+201$, $225-(38+112)$.

Any of the following is called an expression :

(i) A number as 7, 512, 225, 11156

(ii) The sum or the difference of two numbers, *i.e.*,

$3+4, \quad 8-3$

(iii) A group of three or more numbers having the signs of addition or subtraction as : $225-38+112$ or $15-(5+3)$

Brackets

Parentheses are symbols of grouping the numbers and indicate how the numbers are to be combined. Brackets are frequently helpful in grouping the numbers properly.

Operations inside the ( ) are performed first and then the ( ) is treated as a single number, *e.g.*,

$15-(3+5)$

$=15-(8)$..............the numbers 5 and 3 inside the brackets are added first

$=15-8$ ( ) is treated as a single number

$=7$

If we perform all the operations indicated in the expression, we get the number which is called the *value of the expression.* Thus 7 is the value of expression $15-(5+3)$.

56. Find the values of the following expressions :

(a) $2000-(612+588)$ (b) $976+(557-333)$

57. Compare the values of the following expressions :

(a) $44409-(1338-898)$ and $689+(888+1423)$

(b) $36825-(255+681)$ and $681+268252+99999$

58. Write the following (using the signs + or —) :

(a) sum of the numbers 57 and 75

(b) difference of the numbers 86 and 64

59. Prove the following inequalities :

(a) $(85+991)>(83+991)$

(b) $(805+67)>(801+67)$

(c) $(720-326)>(620-326)$

What conclusions do you draw from the above inequalities ? You will find that the **inequality remains unaltered when a natural number is added or subtracted from both the sides of the inequality.**

60. 75 metric tonnes of grain is being transported to the godown and 50 metric tonnes are being taken away from it every hour. Is the quanity of grain in the godown increasing or decreasing ? And at what rate ?

61. A motorist and a cyclist are approaching each other. Their speeds are 40 km/h and 12 km/h respectively. At what speed are they approaching each other ?

62. A motor-boat has overtaken a raft moving in a river with a speed of 6 km/h. The speed of the motor-boat is 15 km/h. What is the speed of the motor-boat with respect to the raft ?

63. A goods-van contains 37 tonnes of goods. At one of the stations 18 tonnes are unloaded and 12 tonnes loaded. How many tonnes of goods are there now in the goods-van ? Work out two expressions for calculating the result. Compare the expressions.

64. Find the values of the expressions given below :

(a) $975-462+562$ (c) $42617+(8000-2617)$

(b) $1219-(219+808)$ (d) $4005-(2495-995)$

65. Write down the following expressions in the statement form :

(a) $305+402$ (c) $2444-0$

(b) $402-305$ (d) $47-47$

66. Write the following as expressions :

(a) the sum of the numbers 0 and 269

(b) the difference of the numbers 488 and 437

67. Prove the following inequalities :

(a) $(695+384)<(695+394)$

(b) $(695+216)>(695-216)$

68. Compare the expressions :

(a) $4378+2562$ and $3267+3675$

(b) $87605-26980$ and $3872+2909$

6. Equations

We know that expressions $15-7$ and $6+2$ represent the same numbers. Two expressions which represent the same number form an equality.

e.g., $15-7=6+2$ or $4+5=9$

Sometimes we are required to use a letter such as x, y or n to represent the numbers. For example, if we are asked to find a number which when added to 14 gives 26, we shall translate this into a mathematical sentence like this :

Let x represent the unknown number.

When x is added to 14, we get $x+14$.

But this is given as 26.

$\therefore\ x+14=26$

Try to guess the number x.

69. Translate the following into mathematical sentences and guess the unknown number :

(1) the sum of two numbers is 25 and one of them is 14.

(2) the difference of two numbers is 18 and the greater number is 42.

(3) if 15 is subtracted from a number, the remainder is 52.

In all the above examples we have an unknown number. An equality which has at least one unknown number is called an **equation**.

Finding the value of the unknown number that will make the equation true, is known as solving the equation.

70. What is the value of the unknown number in the following equations ? Try to guess.

(a) $25+x=40$ (b) $62-x=30$

(b) $x-20=55$ (d) $30=x+18$

71. Find the value of x in the following equations. Try to guess.

(a) $x+x=36$ (c) $5-x=x+1$

(b) $62-x=30$ (d) $x+x=x+5$

Problem 1.

A number is subtracted from 40. The value obtained is equal to the sum of the number and 30. Find the number.

Solution :

Let us denote the unknown number by x. When we write the problem as an equation, we get :

$$40-x=x+30$$

The expression $40-x$ is called the Left Hand Side (L.H.S.) of the equation. The expression $x+30$ is called the Right Hand Side (R.H.S.) of the equation.

Let us try to find the unknown number.

Let us try $x=2$.

$40-x=40-2=38$

$30+x=30+2=32$

L.H.S. of the equation is not equal to the R.H.S. So $x=2$ does not satisfy the equation.

Let us try another number, say, $x=5$.

When $x=5$, L.H.S. $=40-5=35$ and R.H.S. $=30+5=35$.

$\therefore$ L.H.S $=$ R.H.S.

Therefore, $x=5$ satisfies the equation.

Remember that each equation represents the data of a problem. For example, the equation $38-x=29$ represents the following problem.

"What number should be subtracted from the number 38, to get the number 29" ?

72. Translate into sentences :

(a) $32+x=70$ (c) $x+50=83$
(b) $96-x=46$ (d) $x-25=38$

73. Translate each of the following sentences into an equation :

(a) the number x increased by 2 is equal to 6.
(b) 3 subtracted from x is equal to 2.
(c) what number should be subtracted from 302 to get 30 ?
(d) what number should be added to 405 to get 555 ?
(e) from what number have you to subtract 666 to get 261 ?
(f) what should be added to 216 to get 261 ?

74. You have the equality $3+2=5$. Replace the number 2 by the letter x and answer : How to find the unknown term ?

Example :

Solve the equation $x+14=26$

The sum of two addends is 26. One of the addends is 14. To find the other unknown addend x, we subtract the given addend from their sum :

$x=26-14$

$x=12$

To check the solution, we replace x by 12

Then the L.H.S. of the equation $=12+14=26$

The R.H.S. of the equation $=26$

$\therefore$ L.H.S. = R.H.S.

$\therefore$ $x=12$ is correct.

75. Solve the equation $x-18=27$ and check your answer :

76. Solve the following equations :

(a) $768+x=2763$ (d) $x-908=920$
(b) $a+312=564$ (e) $3268=a+889$
(c) $2000-a=1584$ (f) $4222=6111-x$

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Problem 2.

From a bar, a length of 12 metres is cut off and rest of the length measures 27 metres. Find the original length of the bar.

Solution :

Let us suppose that the original length of the bar is equal to x metres.

Length of the bar, when 12 metres is cut off from it, $=x-12$ metres. But this is given as equal to 27 metres.

$\therefore\ x-12=27$

We know how to find the unknown minuend.

$\therefore\ x=27+12=39$

∴ The original length of the bar is 39 metres.

77. Solve the following problems with the help of equations :

(a) From a certain length of wire, 2 metres is cut off. The remaining length is 25 metres. Find the length of the wire.

(b) There are some mangoes in a basket. After adding 37 mangoes to the basket, the total number of mangoes is found to be 75. How many mangoes were there in the beginning ?

78. Using the fig. 1.11, form an equation and solve it.

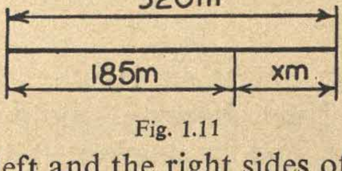

Fig. 1.11

79. Read the left and the right sides of the equations :

(a) $x+782=x+x+18$ (b) $48-x=x+x+x$

80. Solve the equations (guess the number x) :

(a) $x+x+x+x=100$ (c) $25-x=15+x$

(b) $x+x+x=20+x$ (d) $1-x=1+x$

81. Solve the equations :

(a) $x-20489=32511$

(b) $4000015+a=62357684$

82. Solve the following problems using equations:

(a) A boy had 81 paise. He bought an ice-cream and was left with 63 paise. How much did the ice-cream cost ?

(b) A girl gave a milkman a coin of 50 paise for the milk she had bought and got back 22 paise. How much did the milk cost ?

83. Using the figure 1.12, form the equation and solve it.

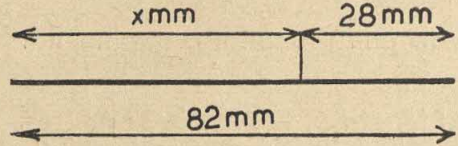

Fig. 1.12.

84. Find the unknown number x having 9 in the units place if:

(a) $208 < x < 210$ (b) $82 < x < 98$

85. What digit must be put in place of the star to make the inequalities true ?

(a) 4*5<415 (c) 738<73*

(b) 334*>3347 (d) 6020<60**

86. Can there be a number such that it is more than 40 and less than 20 ?

87. 6 books were bought in Hindi, Tamil and Urdu. The number of Hindi books is more than the number of Tamil books, and the number of Urdu books is less than the number of Tamil books. How many books were bought in Hindi, Tamil and Urdu respectively ?

88. When is the equality $3+x=3$, true ?

89. Can the sum of two numbers and these numbers end by the same digit ?

90. Can the difference be equal to the minuend ?

91. A number increased by one is 5 less than 20. Find this number.

92. Mother is 42 years old add her daughter is 11 years old. How old was mother when her daughter was 3 years old ? How old will the daughter be, when the mother is 60 years old ?

93. Write down, using each of the digits 1, 2 and 3 only once, all the possible three-digit numbers and place them in the increasing order.

94. Starting from number 2, write down a row of numbers in which each number is 8 more than its preceding number. What number is in the 10th place ?

95. (a) For what value of a, the equality

    $5-a=0$ is true ?

    (b) When does the difference of two numbers end by the digit 0 ?

    (c) When does the difference of two three-digit numbers end by two zeros ?

96. Can a sum of two numbers be equal to

    (a) one of the addends,

    (b) each of the addends ?

97. A father is 32 years old, while his son is 9 years old. How old will the father be when his son is 32 ? How old will be the son, when the father's age increases by 10 years ?

7. Multiplication

You have learnt multiplication in the earlier classes. When we multiply two numbers, we get some number which we say, is the product of the two numbers. In the expression 5×2, 5 is called the *Multiplicand* and 2 is called the *Multiplier*. They are both called *Factors*.

98. Find the product of the smallest three-digit number and the largest four-digit number.

99. Is multiplication, like addition, commutative ?

    Do you get that $323 \times 875 = 875 \times 323$?

Commutative Law of Multiplication

The commutative law of multiplication states that the order of the factors does not affect the product. So, the commutative law of multiplication can be written in the following way :

$$a \times b = b \times a$$

100. Can the product of two numbers be equal to
    (a) one of them,
    (b) each of the numbers ?

101. Are the following statements correct ?
    (a) If one of the two numbers is equal to 1, then their product is equal to the other number.
    (b) If the product is equal to one of the factors, then the other factor is equal to 1.

The number 1 is a special number in several ways. It is the smallest of the counting numbers. We can build any number, no matter how large, by beginning with 1 and adding I's until we have reached the desired number, for example,

$$11 = 1+1+1+1+1+1+1+1+1+1+1$$

Also, when we multiply any number by 1 we get the same number.

102. What do we get when we multiply any number by zero ? Work out these problems and see what you get :
    (a) 0×5 (b) 195×0 (c) 0×357 (d) 0×0

    When we multiply any number by zero we get zero as the answer.

103. One book costs 67 paise. How much should be paid for 43 books ?

104. Write down the following, using the sign of multiplication:
    (a) product of the numbers 745 and 816
    (b) product of the numbers 0 and 326

105. Write the following in words :
    (a) 78×86 (b) 1×327 (c) 0×615

Associative Law of Multiplication

106. Is multiplication, like addition, associative? Do $(4\times3)\times5$ and $4\times(3\times5)$ give the same product ?

Do you see that the product of three numbers is the same whatever be the two numbers you multiply first. Is it also true for more than three numbers ? Try it. Three ways are shown for finding the product of the numbers 4, 5, 6 and 7. They are :

(i) $(4\times5)\times6\times7$
(ii) $5\times(4\times6)\times7$
(iii) $7\times5\times(4\times6)$

Is the product the same in each case ? The associative law of multiplication states that the grouping of factors in a multiplication does not affect the product. If a, b and c are the numbers, then

$$(\mathbf{a}\times\mathbf{b})\times\mathbf{c}=\mathbf{a}\times(\mathbf{b}\times\mathbf{c})$$

107. Show that the following statements are true :

(a) $3\times(11\times14)=(3\times11)\times14$
(b) $23\times(123\times395)=(23\times395)\times123$
(c) $(340\times521)\times60=340\times(521\times60)$

108. Write down the following sums as products :

(a) $7+7+7$
(b) $202+202+202+202$
(c) $k+k+k$
(d) $a+a+a+a+a+a$

In writing the products as $x\times3$ or $a\times6$ where one of the factors is a letter, the numerical factor is always placed before the letter and the sign of multiplication is not written. For example, we write $3x$ for $3\times x$. The numerical factor is called *numerical coefficient* of the letter.

109. Write down the following sums as products :

(a) $a+a+a+a+a+a+a+a$
(b) $k+k+k+k$

110. Represent each of the following products as a sum in two ways :

(a) 3×4 (b) 6×5 (c) 2×8

111. Write down the following products as sums :

(a) $5x$ (b) $3k$ (c) $8a$

112. Find the value of 507a when $a=702$

113. Find the values of the following expressions :
(a) 507a (b) $507+a$ (c) $507-a$
when $a=220$

114. In the following sums dots have been placed instead of the omitted addends. The number of addends is indicated at the top.

$$\text{For example, } \overset{\text{9 addends}}{2+2+2+\ldots\ldots\ldots\ldots 2}$$
$=2+2+2+2+2+2+2+2+2$

What do the the following expressions mean ?

(a) $\overset{\text{10 addends}}{3+3\ldots\ldots\ldots\ldots+3}$ (b) $\overset{\text{12 addends}}{x+x\ldots+x}$

115. For what values of a are the following equalities and equations true ?
(a) $3a=3$ (c) $3a=0$ (e) $3a=a\times3$
(b) $3a=6$ (d) $0\times a=0$ (f) $a\times a=a$

116. With what digit does each of the following products end ?
(a) $1\times2\times3\times4\times5\times6\times7$
(b) $14\times24\times34\times44\times54$

117. Express the following numbers as the product of two equal numbers :
16, 81, 144, 625, 1000

118. Can you express the numbers 25, 40, 400 and 1000 as the product of two equal factors ?

119. Express the number 12 as :
(i) the product of 2 factors
(ii) the product of 3 factors

120. Can you express the number 12 as a product of 4 factors, each of them being a counting number ?

Sometimes the multiplication of two numbers becomes easy, if one of the numbers is factorized into simpler numbers, for example, to find the value of 750 × 8, we work as follows :

$$750\times8=750\times2\times2\times2=(750\times2)\times2\times2$$
$$=1500\times2\times2=(1500\times2)\times2=3000\times2=6000$$

Calculate : (a) 75×8 (b) 125×8 (c) 230×4 (d) 250×12

121. Write down the row of numbers beginning from 1, in which each term is 3 times the preceding term. Which term of this row will be just more than 1000 ?

122. Write down the following sentences in the form of equalities :

(a) the number 37 is 13 more than the number 24.

(b) the number 58 is 24 less than the number 82.

123. Write down as equations the following sentences :

(a) the number a is 23 more than the number 50.

(b) the number 600 is 5 times the number a

(c) the number 220 is 30 less than the number a

124. Solve the equations (guess the number x) :

(a) $2x=x+5$ (c) $5x=0$

(b) $x+x=x$ (d) $x \times x=x$

125. Write down the equations corresponding to the data given in the following questions :

(a) what number increased by 7 gives the number 7357 ?

(b) by what number would you multiply 8 to get the number 496 ?

126. Write down the following sum as product :

(a) $702+702+702+702$

(b) $a+a+a+a+a+a+a$

(c) $x+x+\ldots\ldots+x+x$(800 times)

127. Express each of the following products as a sum :

(a) 80×4 (b) $x \times 7$ (c) $11 \times a$

128. Find the values of the following expressions :

(a) $405x$, when $x=88$ (c) $300a$, when $a=0$

(b) $400-x$, when $x=400$ (d) $a+25$, when $a=0$

129. Find the last digit of the product in :

(a) $15 \times 25 \times 35 \times 45 \times 55$

(b) $26 \times 46 \times 66 \times 76 \times 96$

130. Factorize each of the following numbers into three equal factors :

8, 27, 125, 1000000

131. Write down the row of numbers, in which each number is obtained by multiplying the corresponding number of the row of natural numbers by 4.

132. Write down the following sentences in the form of an equality :

(a) the number 38 is 18 more than the number 20.

(b) the number a is 10 less than the number 45.

133. Form an equation for solving the following problem : In a workshop 420 chairs are manufactured per day. In how many days are 8820 chairs manufactured ?

134. There are 100 apple-trees in a garden. Each tree has 50 branches. There are 25 apples on each branch. How many apples are there in the garden ? Solve it in two ways.

Distributive Law of Multiplication

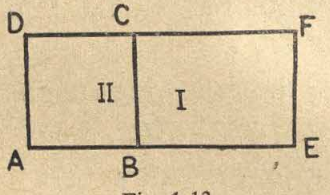

Fig. 1.13

In fig. 1.13 two rectangular fields ABCD and BEFC are shown. The width of the fields is the same and it is equal to EF or BC. You can assume any values for the sides of the fields. For example, let AB=7 m, BE=15 m and BC=EF=8 m. The total area of the two fields=area of AEFD=Length × Width.

$$=AE\times EF$$
$$=(AB+BF)\times EF$$
$$=(7+15)\times 8\text{ sq m}$$

The total area of the two fields can also be found in this way.

The total area of the two fields = area of AEFD

= Area of ABCD + area of BEFD

= length × width of ABCD + length × width of BEFC

= $AB \times BC + BE \times EF$

= 7×8 sq m + 15×8 sq m

∴ we get $(7+15) \times 8 = 7 \times 8 + 15 \times 8$

This equality is very important. Instead of computing the value of the first expression $(15+7) \times 8$, we can find out the value of the second expression $15 \times 8 + 7 \times 8$ and *vice versa.* If we are asked to find the value of an expression, we take that form of the expression of which the value can be found easily.

Let us formulate the rule with the help of which we can write the given expression in the second form.

Look at the equality :

$(15+7)8 = 15 \times 8 + 7 \times 8$

On the left hand side of it the sum of two numbers 15 and 7 is multiplied by the number 8 ; whereas on the right hand side each of the two numbers 15 and 7 is first multiplied by 8 and then their sum is written. Therefore, to multiply the sum by a number, you can multiply each of the addends of the sum by that number and take the sum of the products thus obtained.

This property is called the *Distributive Law of Multiplication.* It can be written in the general form in the following way :

If a, b and c are any three natural numbers then

$(b+c) \times a = ba + ca$

135. Find by two ways the values of the expressions :

(a) $(100+3) \times 305$ (b) $(264+36) \times 25$

136. Suppose you are asked to multiply 76 by 8 orally. You can first multiply 70 by 8, then 6 by 8 and find the sum of two results. Why is the result correct ?

Find orally the products :

(a) 62×6 (b) 412×3 (c) 1008×5

137. Find the numerical values of x and y in the following equations :

(a) $(4+8)\times3=4\times3+x\times3$

(b) $(x+5)\times2=3\times2+5\times2$

(c) $13\times(6+4)=13\times x+13\times y$

(d) $2\times7+3\times7=(x+y)\times7$

138. Using the distributive law, compute the values of :

(a) 93×5 (b) 45×202 (c) 103×65

How to multiply the difference of two numbers by a number ? Try to formulate the rule.

To multiply the difference of two numbers by a number, you can multiply minuend and subtrahend by that number and find the difference of the first and the second products.

139. Apply this rule for finding the values of the following expressions :

(a) $(200-3)\times15$ (b) $(400-4)\times25$

(c) $(300-2)\times20$

140. Find the value of the following expression in two ways :

$(265-65)\times32$

141. To multiply the number 68 by 7 you can first multiply the number 70 by 7 and the number 2 by 7 and then subtract the second product from the first. How did you get the numbers 70 and 2 ? Why is the result correct ?

142. Compute the values of the following using the distributive law :

(a) 99×8 (b) 8×59 (c) 229×25

Example 1. $63\times12=(60+3)\times12=60\times12+3\times12$

$=720+36 \quad =756$

Example 2. $48\times12=(50-2)\times12=50\times12-2\times12$

$=600-24 \quad =576$

Example 3. $75 \times 23 + 25 \times 23 = (75 + 25) \times 23$
$= 100 \times 23 = 2300$

143. Compute the values of the following, using the distributive law :

(a) $69 \times 27 + 31 \times 27$ (b) $263 \times 24 - 163 \times 24$
(c) $603 \times 7 + 3 \times 603$ (d) $65 \times 813 - 613 \times 65$

144. Find the values of the following expressions using the distributive law :

(a) $38x + 62x$, when $x = 283$
(b) $45a + 55a$, when $a = 650$
(c) $375x - 175x$, when $x = 48$
(d) $409a - 309a$, when $a = 79$

145. Write down the expression and then find the perimeter of the rectangle if its sides are equal to :

(a) 25 cm and 8 cm (b) 34 cm and 26 cm

146. Write down the following expressions symbolically and find their values :

(a) the sum of the numbers 864 and 136 multiplied by 308.
(b) the difference of the numbers 3698 and 698 multiplied by 707.
(c) the number 305 multiplied by the sum of the numbers 401 and 104.
(d) the number 305 multiplied by the sum of the numbers 820 and 675.

147. Compute the values of :

(a) $(8 + 12) \times 46$ (e) $25 \times 80 - 1000$
(b) $50 \times (23 \times 42)$ (f) $3200 - 50 \times 4$
(c) $(21 - 9) \times 120$ (g) $60 \times 32 + 80$
(d) $70 \times (43 - 32)$ (h) $1 + 12 \times 13$

148. Check whether the following equality is correct :

$(48 - 28) \times 52 = 800 + 16 \times 15$

149. Find the values of the expressions in two ways :

(a) $(212+288)\times 404$ (b) $(9704-8068)\times 0$

(c) $86\times 303+14\times 303$ (d) $25\times 16-25\times 4$

150. Find the values of the expressions:

(a) $265a+235a$, when $a=38$

(b) $469x-269x$, when $x=150$

151. Form the expression for calculating the perimeter of a rectangle, when its sides are equal to :

(a) 48 cm and 56 cm (b) 400 m and a m

152. Form expressions and find their values corresponding to the sentences given below :

(a) multiply the difference of the numbers 2000 and 1394 by the number 602.

(b) multiply the number 305 by the sum of the numbers 864 and 139.

153. Compare the values of the following expressions :

(a) $(81+9)\times 40$ and $81+9\times 40$

(b) $(82-2)\times 35$ and $82-2\times 35$

9. Volume of a Rectangular Parallelepiped

A match-box, a brick and a wooden box (fig. 1.14) have the

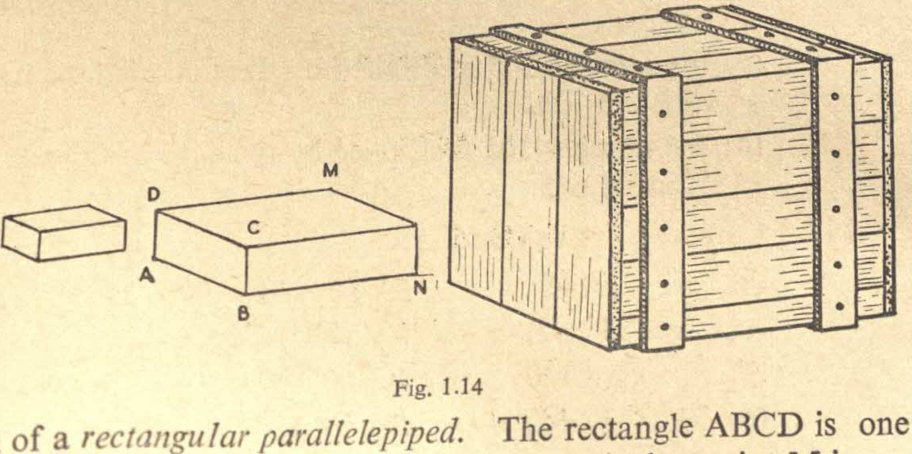

Fig. 1.14

form of a *rectangular parallelepiped.* The rectangle ABCD is one of its faces, segment BN is one of the edges and the point M is one of the vertices of the rectangular parallelepiped.

154. How many faces, edges and vertices does the rectangular parallelepiped in fig. 1.15 have ?

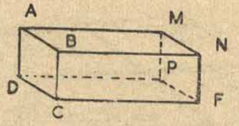

Fig. 1.15

Note : When you look at a solid you cannot see all its edges or faces, unless the solid is transparent. Therefore, the invisible edges are shown by the dotted lines.

155. Which edges in fig. 1.15 have vertex B as common ? How many edges are there in the rectangular parallelepiped ?

Can you show four edges, having a common vertex ?

156. In fig. 1.15 what are the faces which contain the edge DC, the edge MN and the edge PF ? How many edges are there in the face PFNM ? What are the faces which contain the vertex D ? How many vertices lie in the face APDM ?

157. The dimensions of a rectangular parallelepiped are given in fig. 1.16. What is the length of the other edges ? Find the area of the face DCMN.

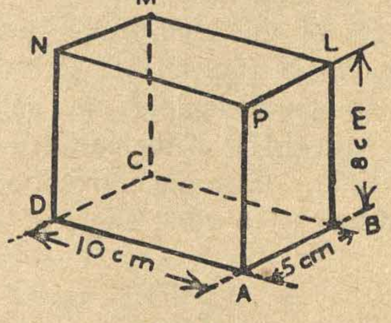

Fig. 1.16

158. How many centimetres of rope will be needed to fasten a parcel, shown in fig. 1.17 (length$=$20 cm, width$=$15 cm,

height=7 cm) ? Ends of the rope should be left loose by 2 cm each.

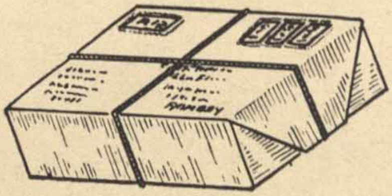

Fig. 1.17

159. What shape has a suitcase in fig. 1.18 ? Find out how many square centimetres will be needed to make a cover for this suitcase (length=7 dm, breadth=4 dm, and height=2 dm) ?

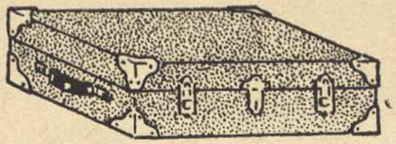

Fig. 1.18

160. A rectangular parallelepiped can be placed on the plane surface by any one of its faces. Therefore, any one of them can be taken as a base. The face EFMN (fig. 1.19) is the base of the rectangular parallelepiped. Name the vertices and faces of the rectangular parallelepiped which do not belong to its base.

Three edges of the rectangular parallelepiped have a common point N (fig. 1.19). Two of them, lying in the base of the parallelepiped, are called the width (EN) and the length (MN) of the parallelepiped and the third, ND the height. Give the corresponding names (length, width and height) to other edges of the parallelepiped.

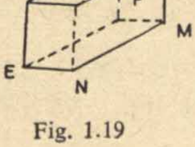

Fig. 1.19

161. Place the model of a rectangular parallelepiped on a table, with different faces as base. Show the length, the width and the height of the parallelepiped in each case.

162. Is it possible to call a cube a rectangular parallelepiped ? Can you name a rectangular parallelepiped as a cube ?

163. How many small cubes do you need to construct a big one shown in fig. 1.20 ?

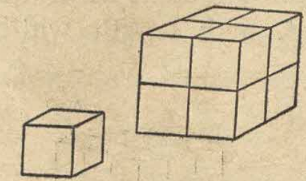

Fig. 1.20

164. How many cubes would you need to construct a parallelepiped shown in fig. 1.21 ?

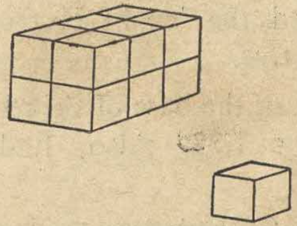

Fig. 1.21

A cube whose edge is 1 cm. long is called a centimetre cube. How many cubic centimetres are there in a big cube in fig. 1.21 ?

165. How many one centimetre cubes can you place (in one laying) on the rectangle of which the dimensions are ?

(a) 8 cm and 2 cm (b) 20 cm and 11 cm

(c) 40 cm and x cm

166. How many centimetre cubes can you place one over the other on the rectangle of which the length and width are 10 cm and 8 cm, to get a rectangular parallelepiped, with a height 11 cm ?

167. How many cubic centimetres can be cut off from a rectangular parallelepiped, with the following measurements ?

(a) length $=20$ cm, width $=15$ cm, height $=2$ cm

(b) length $=a$ cm, width $=8$ cm, height $=2$ cm

The number, showing how many cubic centimetres there are in a rectangular parallelepiped, is called its volume (in cubic centimetres).

168. Find the volume (in cubic centimetres) of a rectangular parallelepiped, with length width and height equal to 3 cm each.

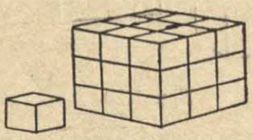

Fig. 1.22

169. The area of the base of a rectangular parallelepiped is 20 sq cm and the height is 6 cm. Find its volume in cubic centimetres.

170. Find the area of the base of the rectangular parallelepiped shown in fig. 1.23. Also, find its volume in cubic centimetres.

Formulate the rule of calculating the volume of a rectangular parallelepiped.

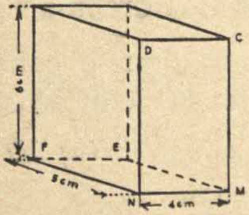

Fig. 1.23

Volume of a rectangular parallelepiped (in cubic centimetres) is equal to the area of the base (in square centimetres) multiplied by the height in centimetres.

Take the model of a rectangular parallelepiped and also its measurements to find its volume (in cubic centimetres).

171. The volume of a rectangular parallelepiped is equal to 24 cu cm. Find its length, width and height. Is there only one rectangular parallelepiped having the volume equal to 24 cu cm ?

Write down the answer as : $24 \times 1 \times 1$ cm^3

172. Find the volume (in cubic centimetres) of the following rectangular parallelepiped, if the edges in centimetres are :

(a) 100, 10, 24
(c) 80, 4, 60
(b) 30, 20, 30
(d) 20, 20, 20

173. Find the volume (in cubic centimetres) of a cube, an edge of which is equal to :

(a) 3 cm (b) 5 cm (c) 30 cm

174. (a) Make a paper model of a rectangular parallelepiped, using its development shown in fig. 1.24. Show its vertices, edges and faces.

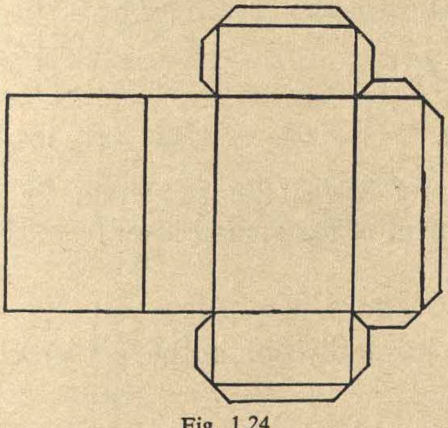

Fig. 1.24

(b) Prepare the model of a rectangular parallelepiped, which has length equal to 4 cm, width 3 cm and height 2 cm.

175. Find the lengths of the edges and areas of the faces of the rectangular parallelepiped shown in fig. 1.25.

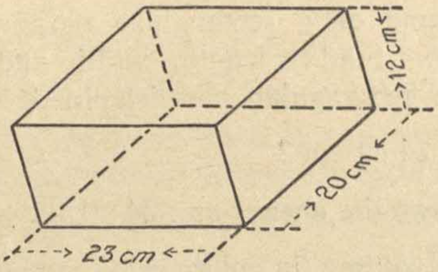

Fig. 1.25

176. In fig. 1.26 is given a wire model of a rectangular parallelepiped. Show all the possible ways of going along its edges from the point F to the point C. Are these routes equal in length ? Can you show two routes of unequal lengths ?

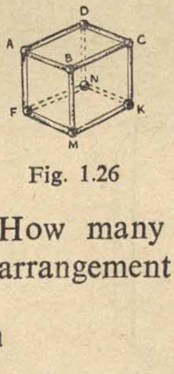

Fig. 1.26

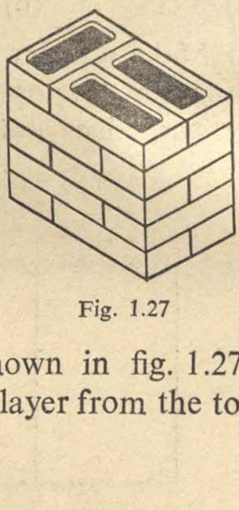

Fig. 1.27

177. How many bricks are shown in fig. 1.27 ? Draw the arrangement of the second layer from the top.

10. Division

You must have learnt division in the earlier classes. Study the following examples.

178. During the visit to a zoo, 25 boys and girls wanted to go for an elephant ride. If an elephant could carry 5 people, how many elephants were needed ? How many 5's are there in 25 ?

179. If I had spent Rs. 40/- to buy 8 fountain-pens, how much did I pay for one fountain-pen ?

In division we find out how many equal groups of a particular size are contained in another group or we divide a given number into a given number of equal parts and find the number of such parts.

Below are given the definitions of certain technical terms being used in division.

The number which is divided is called *Dividend* (*e.g.*, 40); the number which divides is called *Divisor* (*e.g.*, 8) and the number got on dividing is called *Quotient* (*e.g.*, 5).

You know that 40 divided by 8 gives 5 as quotient.

You also know that $8 \times 5 = 40$.

Therefore $40 \div 8 = 5$ means that $5 \times 8 = 40$

Fill in the gaps :

(a) Dividend divided by...............gives quotient.

(b) Quotient multiplied by.........equals dividend.

(c) Dividend divided by quotient equals............

180. Do you think that division is opposite to multiplication ? Use this idea to find a check for division. Can you suggest another check for division ?

181. There are 12936 cubic centimetres in 42 congruent rectangular parallelepiped. Find the volume of each of the parallelepiped.

182. How many times would you subtract 125 from 1000 so that nothing is left ?

183. Can you always subtract one number from another number ? Can you always divide one number by another number and obtain a quotient ?

184. You have seen that multiplication is related to addition. Can you find out any relationship between subtraction and division ?

185. Is division commutative and associative ? Give examples.

You know that when we multiply any number by 1 we get the same number. What happens if we divide any number by 1 ?

Is this true that $1 \times a = a \times 1$ where a is any natural number ?

Can we say the same thing for division ?

Is $1 \div a = a$ true where a is a natural number ? Is it always possible ? Guess the value of a, for which it is true.

Let us see if zero follows the rules for division of counting numbers.

If I have no money in my pocket and I distribute it equally among 5 boys, each boy will get no money

$$\therefore \quad 0 \div 5 = 0$$

So, zero divided by any natural number always gives zero. Can we say that division of zero by a natural number is never a natural number ?

Division by Zero is Impossible

Does the above statement agree with the definition of multiplication by zero ?

We have earlier defined an expression for two operations of addition and subtraction. We can extend the same definition for all the four operations that we have learnt so far. Thus, any one of the following is an expression :

(i) a number as 7, 1232, 452

(ii) an indicated sum, difference, multiplication or division of two numbers as :

$$5+2,\ 122-85,\ 5 \times 12,\ 42 \div 6$$

(iii) a group of three or more numbers using one or more operations as :

$452-(42+122),\ 3 \times 5+(144 \div 12)$

We use parentheses, as before, to indicate how numbers are combined.

186. Find the values of the expressions :
(a) 72315÷45 (b) 328032÷804
Check the answer in two ways.

187. How will the number, ending by two zeros, change if these zeros are crossed out ? Give an example.

188. Make division :
(a) 4700÷100 (b) 703000000÷1000000
If the dividend and the divisor are multiplied or divided by the same number simultaneously, the quotient will not be changed.

189. Use this main property of a quotient for the division in the following exercises :
(a) 364800÷600 (b) 617130000÷240

190. Express the number 2431100 as a product of two factors, one of which is 302.

191. Is it possible to factorize the number 9605 into two factors, one of which is 28 ?

192. Which of the numbers 4, 7, 16, 21, 22, 805, 908, 24330 and 48789 are divisible by 2 ?

Numbers which are divisible by two are called *even* and those which are not divisible by 2 are called *odd.* By what digits do the even numbers end ? By what digits do the odd numbers end ?

193. Find out the rule according to which the row of numbers given below, is formed :
1, 3, 5, 7, 9, ...
What will be the number in the 10th place ? Does this row include at least one even number ?

194. Write down the row of numbers which are divisible by 3 and write them in the increasing order. What will be the number in the 25th place ?

195. What kind of number (odd or even) will be the sum, difference and product of
(a) two even numbers (b) two odd numbers
(c) an even number and an odd number ?

196. Write down the following phrases in the form of an expression :

(a) the quotient of the number 500 divided by 100.
(b) the quotient of the number 0 divided by 335.

197. Write the following expressions in words :

(a) $303303 \div 101$ (b) $88008 \div 88008$ (c) $0 \div 568$

198. A car consumes 210 litres of petrol weekly. For how many days will 1470 litres of petrol be sufficient ? Solve this in two ways, working out expressions.

199. Find the value of the expression $21672 \div x$, when

(a) $x=516$ (b) $x=42$

200. Starting from the number 2048, write the row of natural numbers, in which each preceding term is twice the succeeding term. Will you get the number 413 in this row ? Has this row an end ?

201. Write down (in increasing order) the row of even numbers which are divisible by 5. What is the rule according to which this row of numbers is formed ? What will be the number in the 12th place ?

202. Starting from the number n, write the row of numbers in which each term is more than its previous term by n. State the rule according to which this row of numbers can be formed from the row of natural numbers. What will be the number in the 100th place ?

203. The number 7680 is divisible by 24. Find two numbers nearest to the number 7680, which are also divisible by 24.

204. Each almirah contains 5 shelves. We can place 34 books in each shelf. How many almirahs are required to place 3740 books ? Solve this in two ways.

205. Write down the following statements in the form of equalities :

(a) the number 84 is four times the number 21.
(b) the number a is 20 times the number 8.
(c) the number a is 20 less than the number 300.

206. Write down the following statements in the form of inequalities :

(a) the sum of the numbers 31 and 65 is more than the number 50

(b) the number 80 is less than the difference of the numbers 336 and 97

(c) the product of the numbers 46 and 45 is less than the number 4000

(d) the number 72 is more than the the quotient of the numbers 800 and 80

207. Look at the equality : $2 \times 3 = 6$

Replace one of the factors by the letter x and answer how to find the unknown factor when the product is known.

208. How will you find the value of the unknown in the following ?

(a) $262\,x = 116590$ (b) $51357 \div a = 57$

(c) $a \div 301 = 803$

Check the answers you have got in the above exercises.

209. Solve the following problems by the method of equations :

(a) by what number is the number 33966 to be divided to get the number 333 ?

(b) the volume of a rectangular parallelepiped is equal to 840 cm^3. If its height is 4 cm, find the area of the base.

210. Solve the equations :

(a) $3x + 5x = 816$ (b) $8a - a = 637$

211. Write each of the following statements in the form of an expression :

(a) divide the difference of the numbers 20000 and 13030 by 34.

(b) subtract the quotient of the numbers 154 and 11 from 163.

(c) add the quotient of the numbers 154 and 11 to the number 4031.

212. Write down the following statements in the form of an equality or inequality as the case may be :

(a) the product of the numbers 46 and 88 is more than their sum.

(b) the product of the numbers 0 and 68 is equal to the quotient when 0 is divided by 68.

(c) the difference of the numbers 408 and 1 is less than the quotient when 408 is divided by 1.

213. The volume of a rectangular parallelepiped is equal to 405 cm^3. Find the volume of another rectangular parallelepiped with the same height, but its base being 5 times larger than the base of the given parallelepiped.

214. Simplify the following :

(a) $(47-38)\times 10+87$ (b) $25\times(110-90)-90$

(c) $840\div(27+15)+20$ (d) $70+30\div(301-286)$

215. Five points are marked on a straight line. Find the number of the segments having continuous points as their end points.

216. There are P points marked on the boundary of a circle. Find the number of arcs having continuous points as their end points.

217. Prove that the following inequalities are true :

(a) $3876\div 57<3990\div 57$

(b) $4370\div 45>4370\div 95$

218. Find the values of the following :

(a) $120a\div 40$ (c) $m\div m$

(b) $25x\div 25$ (d) $4m\div 4$

219. The sum of three products formed by the numbers 3, 5 and 12 by an unknown number is equal to 480. Find the unknown number.

220. The difference between the products got by multiplying each of the numbers 3 and 8 with an unknown number is 105. Find the unknown number.

221. The product of an unknown number and 10 is greater than the product of the unknown number and 6 by 240. Find the unknown number.

222. A table and a chair together cost Rs. 40/-, the table costing 9 times the chair. What are the prices of the table and the chair ? Solve by the method of equation.

223. Two adults and three children have paid 90 paise for the tickets to visit an exhibition. What is the price of the ticket for an adult, if it is three times that for a child ?

224. The perimeter of a rectangle is equal to 490 m. Find the length and breadth of the rectangle, if its length is 6 times its breadth.

225. The sum of three numbers is 820. The second number is 5 times the first one, and the third number is 7 times the second one. Find what these numbers are.

226. The minuend in a subtraction sum is 7 times the difference. What multiple is the subtrahend of the difference ?

227. The subtrahend is 12 times the difference. Find the subtrahend and the minuend, if their sum is equal to 75.

228. The sum of two numbers is 27. The greater of them is twice the difference of these numbers. Find these numbers.

229. The sum of five successive natural numbers is 6015. Find the smallest of these numbers.

230. The sum of six successive odd numbers is 36516. Find the greatest of these numbers.

231. Compute the volume of a cube, if its edge is equal to :
(a) 3 dm (b) 1 dm

The volume of a cube, whose edge is equal to 1 decimetre is 1 *cubic decimetre*. In one cubic decimetre there are 1000 cubic centimetres (1 cubic decimetre=1000 cubic cm).

232. Find the volume (in cubic centimetres) of a rectangular parallelepiped, whose dimensions in dm are :
(a) 4, 15, 7 (b) 25, 11, 8

233. Find the volume (in cubic decimetres) of a cube, whose edge is equal to :

(a) 4 m (b) 1 m

234. The volume of a cube, whose edge is equal to one metre, is 1 cubic metre. In one cubic metre there are 1000 cubic decimetres (1 cu m=1000 cu dm). Find the volume (in cubic decimetres) of a rectangular parallelepiped whose dimensions in metres are :

(a) 7, 6, 2 (b) 12, 13, 2

235. How many cubic centimetres are there in one cubic metre ? How many square centimetres are there in one square metre ? How many centimetres are there in one metre ?

236. The area of the first field in fig. 1.28 is 17000 m². Find the area of the second field.

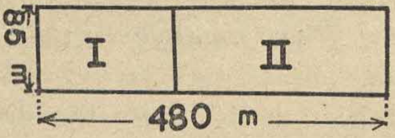

Fig. 1.28

237. Calculate the volumes of the rectangular parallelepiped whose dimensions are given below :

(a) 9 cm, 2 dm, 5 cm (b) 25 dm, 780 cm, 4 dm
(c) 20 dm, 5 m, 6 m

238. Find the value of the expression :
$28x+74x-2x$, when $x=308, 0$

239. Check the equality :
$8010-88110\div22=5+32\times125$

10. Division with a Remainder

The number 96 is divisible by 8 because $12\times8=96$. The number 57 cannot be divided by 6, because $9\times6=54$. So, in the number 57 the number 6 is contained 9 times and 3 is left. We can express it as: $57=9\times6+3$. The number 57 is called the *Dividend*, the number 6 is called the *Divisor*, the number 9 is called the *Quotient* and the number 3 is called the *Remainder*. The remainder is always *less* than the divisor. Writing down the equalities, check whether the following table has been prepared correctly.

Dividend	Divisor	Quotient	Remainder
1220	48	25	20
638	8	79	6
632	9	70	2

240. Find the remainders obtained on dividing each of the numbers 4865 and 4901 by 29.

241. Write down three numbers which when divided by 25 give a remainder equal to 24. Can you find some more such numbers ?

242, Squares are to be cut out of the sheet of a paper, 950 mm by 1200 mm. The sides of the squares can be either 46 mm or 64 mm. In which case the quantity of waste paper will be minimum ?

243. A particular year started with Friday. From what day of the week would the next year begin ?

Example

Using the table given below, find the remainder when the following expressions are divided by 21.

(a) 3796+5170
(c) 3964−3796
(b) 4805−3796
(d) 4805+3796

Use the remainder of the sum equal to the sum of the remainders if this sum is less than 21. If it is greater than 21, divide the sum of the remainders and get the new remainder, which will be equal to the remainder of the sum. Similary we can find a rule for the difference.

Number	Remainder after dividing by 21	Number	Remainder after dividing by 21
3796	16	4805	17
3964	16	5170	4

244. There are 32 seats in a row. How many rows will be occupied by 475 pupils ?

245. Compute :
    (a) $(448+952)\div28-952\div28$
    (b) $(3074-2485)\times307+307\times2485$

246. Are the following statements true ?
    (a) If the sum of two numbers is odd, the difference of these numbers is also odd.
    (b) If the difference of two numbers is odd, the sum of these numbers is also odd.
    (c) If the sum of two numbers is odd, their product is also odd.
    (d) If the product of two numbers is odd, the sum of these numbers is also odd.

247. The product of 4 numbers is an odd number. Will their sum be odd or even ?

248. The product of 5 numbers is an even number. Will their sum be odd or even ?

249. A school bought note-books equal to 5 times the number of the text-books. One note-book costs Re. 1/- and one text-book costs Rs. 4/- ; how many books were bought if the total amount spent was Rs. 252 ?

250. Two trains leave stations A and B 780 km apart at the same time. They move in opposite directions to meet each other. One train is moving at a speed of 53 km/h and the other at 77 km/h. What will be the distance between the two trains after 2 hours and 6 hours respectively ?

251. Find the values of the expressions :

(a) $2a+x$, when $a=405$ and $x=8000$

(b) $2(a+x)$, when $a=405$ and $x=8000$

(c) $(a-b)\div47$, when $a=4465$ and $b=2585$

(d) $a-b\div47$, when $a=4465$ and $b=2585$

252. Find the values of :

(a) $7770-770\div(43+27)$

(b) $(480-48)\div24+18$

(c) $8(23+37)-9\div(62-59)$

(d) $(16+14)\times42+58\div(60-31)$

253. A man and a boy started walking at the same time from a town. The man walks at a speed of 6 km 500 m/h and the boy at a speed of 4 km/h. Both of them travelled in the same direction. By what distance was the boy behind the man in an hour ? What would be the distance between them after three hours ?

254. A square is divided into 9 squares. Write the numbers 0, 1, 0, 3, 0, 5, 6, 7 and 8 in these 9 squares such that the product of the numbers in each column and each row would be the same ?

255. Find the values of the following expressions :

(a) $a\div b-468$, when $a=142740$ and $b=305$

(b) $7960-2982\times(a-b)$, when $a=b$

256. Find the sum of three numbers if the first number is a, the second is more than the first by 38 and the third is 3 times the second. Find the value of this sum when $a=52$.

257. There are x pupils in class IV. In class XI the number of pupils is 15 more than that in class IV and the number of pupils in class V is twice that of in class XI. How many pupils are there in all the three classes together ?

258. Find the values of the expressions :

(a) $34ab+48ab+56ab-38ab$ when $a=16$, $b=17$

(b) $23kn-9kn+36kn$ when $k=15$, $n=84$

259. Let us suppose that all the even numbers are written in the increasing order. Then the number a of this row can be found with the help of the equality $a+2n$, where n is the number of the term in the row. Find the 41st term of this row.

260. A father is x years old. His son is 33 years younger to him ; mother 3 times older to her son. How old are the father, the mother and the son ? Can x be less than 33 ? Solve the problem when $x=48$.

261. Find the values of the expressions :

(a) $8ab+8ac+8ad$, when $a=2$, $b=3$, $c=5$ and $d=6$

(b) $5ma-3mb+6mc$ when $m=3$, $a=5$, $b=1$ and $c=0$

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CHAPTER II

INTEGERS

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1. Positive Integers, Negative Integers and Zero

1. A squirrel is running up and down a tree-trunk. Show on fig. 2.1 where she is if the distance between her and the hole (the point O) is equal to 3 metres. (scale 1 division=1 m)

2. To find the position of the squirrel, it is not sufficient to know only this distance of the squirrel from O. It is also necessary to know the direction in which the distance has to be measured. Mark on fig. 2.1 the position of the squirrel if she is :

   (a) 2 metres above the hole

   (b) 3 metres below the hole

   (c) 5 metres above the hole

   (d) 1 metre below the hole

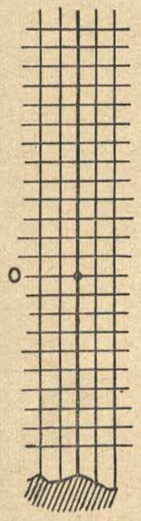

Fig. 2.1

3. Such expressions as ‘2 metres above the hole’ or ‘3 metres below the hole’ can be written in a short form as :

   ‘a 2 m’ or ‘b 3 m’.

   Point out the place where the squirrel will be if she is :

   (a) a 1 m (b) b 1 m (c) b 0 m (d) a 0 m

4. One squirrel is b 7 m, and the other is b 42 m from O. Which of them is higher ?

5. Three squirrels are at the points A, B and C respectively (fig. 2.2). Which of them is

   (a) above all the others,

   (b) below the others,

   (c) between the other two ?

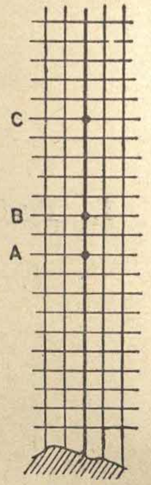

Fig. 2.2

6. A road running south to north passes through a camp at O (fig. 2.3). A group of cyclists left the camp and covered the distance of 4 km on this road. Mark the position of the group on fig. 2.3 (scale 1 division = 1 km).

   To find the position of the group, we have to know not only the distance but also the direction (towards south or north) in which the group has moved. Show on fig. 2.3, where the group of the cyclists is when it is :

   (a) 5 metres to the north of the camp

   (b) 2 metres to the south of the camp

   (c) 3 metres to the south of the camp

   (d) 7 metres to the north of the camp

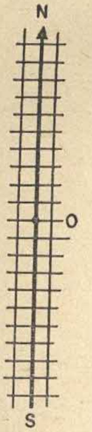

Fig. 2.3

If we take some new examples, we have to introduce new pairs of signs, for example :

$l\,5^\circ$ (5° lower than 0°) and $h\,6^\circ$ (6° higher than 0°). But we know that in this case signs '+' and '–' are used, *i.e.*, we write $^{+}6^\circ$ for $h\,6^\circ$ and $^{-}5^\circ$ for $l\,5^\circ$.

Is it possible to use these signs in other cases also, the sign '+' instead of '*h*' and "*n*" and the sign '–' instead of '*l*' and '*s*' ? We write these signs by the side and just above the middle of the number to indicate the direction, so as not to mix them with the signs of addition and subtraction. It is a more economical way of indicating the direction in which the squirrel, or the group of cyclists is moving. To indicate numbers in one direction, well known natural numbers are used ; but for the numbers in the opposite direction new numbers, called *negative numbers,* are used. These numbers are notated as $^{-}5$ and $^{-}6$ (minus five and minus six).

To emphasise the opposite character of the 'old' and 'new' numbers, the 'old' numbers are called *'positive' numbers.* Sometimes the sign '+' is written by their side. So $^{+}4$ and 4 are the same numbers.

Number 0 is neither a negative nor a positive number. Positive numbers, negative numbers and zero form a group of numbers called *Integers.*

7. Do you know any other opposites that sometimes are marked by plus and minus signs ?

   In many activities in life, *opposites* occur.

   What is the opposite of each of the following ?

   (a) Riding north
   (b) Moving backward
   (c) Having a profit
   (d) Increase in price
   (e) Gaining weight
   (f) Going up
   (g) Rise in temperature
   (h) Earning money
   (i) Sailing east
   (j) Excess of rain
   (k) Depositing money in a bank
   (l) Adding
   (m) Above average
   (n) Increasing population
   (o) Above sea-level

8. Write three numbers : (a) positive, (b) negative, and (c) natural. Name the number which is neither positive nor negative.

9. Show on fig. 2.1 the place where the squirrel is, if her distance in metres from the hole is (a) $^{+}6$; (b) $^{-}3$; (c) 4 ; (d) $^{-}2$; (e) $^{-}1$; (f) 0.

10. Show on fig. 2.3 the place where the group of cyclists is, if its position is shown (in km) by the following numbers:

    (a) $^{-}8$ (b) $^{-}6$ (c) 1 (d) $^{+}2$ (e) 0

11. Show on the model of a thermometer where the upper end of the mercury is, when the temperature (in degrees) is :

    (a) $^{-}25$ (b) $^{+}30$ (c) $^{+}12$ (d) $^{-}18$ (e) $^{+}41$

12. Explain the meaning of the phrase : Today the temperature was t°C, when $t = {}^{+}15$; $^{-}3$; 0 ; 44

13. Tell what is represented by :

    (1) $^{-}3$ km if $^{+}3$ km represents a distance towards north

    (2) $^{-}$Rs. 10 if $^{+}$Rs. 5 represents Rs. 5 gained

    (3) $^{-}500$ m if $^{+}1000$ m means going 1000 m up

    (4) $^{-}$Rs. 20 if $^{+}$Rs. 15 means Rs. 15 saved

    (5) $^{-}12°$ latitude if $^{+}5°$ means 5° north latitude

    (6) $^{-}4°$ if $^{+}2°$ means 2° rise in temperature

14. Give your own examples of two opposites.

15. Give two examples of negative numbers and explain their meaning. If the expression A ($^{+}200$) means Town A is 200 m above sea-level, what do B ($^{+}300$) ; C ($^{-}100$) ; D ($^{+}70$) mean ?

16. Draw the sketch of a road passing through a camp (running east-west) and mark points A, B, C and D, the distances (in km) from the camp and the directions are expressed by numbers $^{-}3$, $^{-}8$, $^{+}7$ and $^{+}1$.

17. Write down 3 positive and 5 negative numbers.

2. Comparison of Numbers

We can represent natural numbers on a number-ray (fig. 2.4).

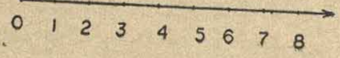

Fig. 2.4

Continue the ray in the opposite direction.

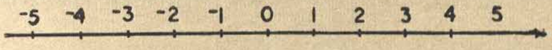

Fig. 2.5

To the left of 0, on this line, mark the segments of lengths equal to 1, 2, 3, 4 units etc. These numbers on the left of the point 0 are negative numbers. They are being represented on the number-line.

18. Mark on a straight line the following numbers :
$^{-}6$, $^{+}3$, $^{-}8$, $^{+}10$, $^{-}11$, $^{+}9$, $^{+}7$, 8. (scale : 1 div. = 1 unit).

19. Mark on a straight line the numbers :
$^{-}10$, $^{-}20$, $^{+}40$, $^{+}60$, $^{-}80$ (scale : 1 div.=10 units).

20. On a straight line numbers $^{+}1$ and $^{-}1$ are marked (fig. 2.6). Mark on this line the following numbers :
$^{+}4$, $^{-}5$, $^{-}7$ and $^{+}10$

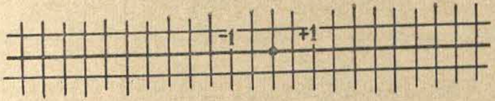

Fig. 2.6

21. Name from a number-line a few of the numbers which are :
(a) to the left of the number $^{+}240$
(b) to the left of the number $^{-}5$

22. Name the 'neighbours' of the following numbers :
$^{+}130$, $^{+}800$, $^{+}210$, $^{-}300$ and 0

23. Write all the numbers on a number-line which are between the following numbers :
(a) $^{+}20$ and $^{+}24$ (b) 0 and $^{+}7$
(c) $^{-}8$ and $^{-}5$ (d) $^{-}2$ and $^{+}2$
(e) $^{-}3$ and 0 (f) $^{-}3$ and $^{+}5$

24. On the number-line which of the numbers are at the same distance from 0 (scale : 1 div.=1 unit) ?

You know that out of two natural numbers the greater number lies to the right of the other number on the number-line.

This rule is true for all integers on the nmmber-line. Of the two integers, the greater number lies to the right of the other.

25. Mark on the number-line :

$0, {}^{+}3, {}^{-}5, {}^{-}3, {}^{+}8, {}^{-}7, {}^{+}1, {}^{-}2, {}^{-}10$ and compare :

(a) 0 and $^{+}9$

(b) 0 and $^{-}5$

(c) $^{+}8$ and 0

(d) $^{-}2$ and $^{+}3$

(e) $^{-}7$ and 1

(f) $^{+}3$ and $^{-}3$

26. Temperature in Nainital is $^{-}5°$ and in Simla, $^{-}10°$. Tell where the temperature is higher.

27. Compare a and b, when :

(a) $a = {}^{-}4; b = {}^{-}1$ (b) $a = 0 ; b = {}^{-}200$

28. Put the sign of inequality > or < instead of a star :

(a) $^{-}240^{*}\ {}^{+}3$ (b) $^{+}96^{*}\ {}^{-}80$

29. Place the numbers $^{+}9, {}^{-}40, {}^{-}13, {}^{-}0, {}^{+}1, {}^{-}7$ and $^{+}7$ in the increasing order, using one of the signs of inequality.

30. Place the numbers $^{-}25, {}^{-}30, {}^{+}29, {}^{-}0, {}^{+}5, {}^{-}2$ and $^{+}1$ in the decreasing order, using the sign of inequality.

31. Let a and b be two positive numbers ; m and n two negative numbers. Compare :

(a) 0 and a

(b) b and 0

(c) m and 0

(d) 0 and n

(e) a and m

(f) m and n

(g) m and b

(h) b and n

(k) a and b

32. Determine whether x is positive, negative or zero when :

(a) $x > 0$ (b) $x < 0$

33. Is 'a' a positive or a negative number if :

(a) $a < b$ and b is a negative number,

(b) $a > b$ and b is a positive number ?

34. What values of x will satisfy the following inequalities?

(a) $^{+}23 < x < {}^{+}25$

(b) $^{-}37 < x < {}^{-}35$

(e) $^{-}2 < x < {}^{+}2$

(c) $^{-}3 < x < 0$

(d) $^{-}1 < x < {}^{+}1$

(f) $^{-}3 < x < {}^{-}1$

35. Is there a number which is :
    (a) greater than $^{+}8$ and less than $^{-}2$,
    (b) greater than $^{-}2$ and less than $^{+}8$,
    (c) less than $^{-}8$ and greater than $^{-}2$,
    (d) greater than $^{-}8$ and greater than $^{-}2$?

36. Compare a and c when :
    (a) $a<b$ and $b<c$ (c) $c>b$ and $a<b$
    (b) $a>b$ and $b>c$ (d) $c<b$ and $a>b$

37. Mark on the number-line the following numbers by taking a suitable scale :
    $^{+}200$, $^{-}500$, $^{+}800$, $^{-}400$, $^{-}100$

38. What numbers are between the numbers ?
    (a) $^{-}46$ and $^{-}41$ (b) $^{-}5$ and $^{+}2$

39. What numbers on the number-line are at a 'distance' of :
    (a) 5 units from the number $^{-}5$,
    (b) 4 units from the number $^{+}4$,
    (c) 3 units from the number $^{-}1$?

40. Compare numbers a and m if :
    (a) $a = {}^{-}590$ and $m = {}^{-}398$
    (b) $a = {}^{-}5050$ and $m = {}^{-}6059$

41. Compose all possible inequalities, using the numbers $^{+}9$, $^{-}5$ and $^{-}7$.

3. Addition

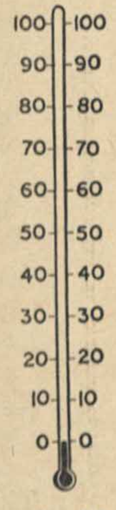

Fig. 2.7

42. Numbers are used to indicate the *increase* and *decrease* in temperature etc. In fig. 2.7 the position of mercury, when the temperature is 0°, is shown. Show where the end of the mercury will be if there is :
    (a) an increase in temperature by 70°
    (b) a decrease in temperature by 50° after the mercury has shown 70° temperature

Example :

Where will you be if you take $^{+}8$ steps and follow that with $^{-}6$ steps, assuming that $^{+}8$ steps means 8 steps forward ?

Solution :

8 steps taken forward, followed by 6 steps backward, the result is 2 steps forward.

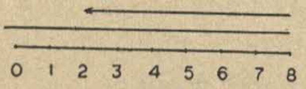

Fig. 2.8

Briefly :

$$(^{+}8 \text{ steps}) + (^{-}6 \text{ steps}) = {}^{+}2 \text{ steps}$$

We shall represent displacements by points on the number-line. If the point moves to the right, it is denoted by a positive number and by a negative number, if the point moves in the opposite direction.

43. Draw a number-line and find where the point A will be on this line if it moves :

(a) $^{+}3$ units (b) $^{-}2$ units (c) $^{-}1$ units
(d) $^{-}3$ units (e) $^{-}5$ units

44. At which point on the number-line will the point M be if it moves from the point 0 :

(a) first by $^{+}5$, then by $^{+}2$, (d) first by $^{+}3$, then $^{-}3$,
(b) first by $^{-}3$, then by $^{-}4$, (e) first by $^{-}6$, then by $^{+}4$,
(c) first by $^{+}4$, then by $^{-}2$, (f) first by $^{-}7$, then by $^{+}9$?

45. With the help of a number-line it is very easy to add positive numbers. If, for example, you first find the number 2 on the number line and move to the right by 6 units, the result will be the sum of 2 and 6 (fig. 2.9).

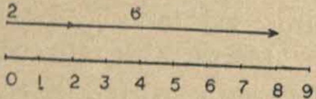

Fig. 2.9

This rule can be used when adding a positive number to any number. For example :

$$^{-}5 + {}^{+}7 = 2 \text{ (fig. 2.10)}$$

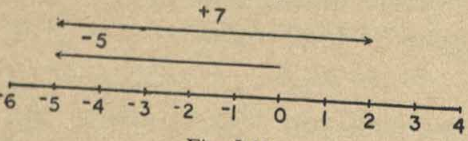

Fig. 2.10

When adding a negative number to a given number, for example, $^{-}4$ is added to $^{+}3$, we move from the given number 4 units

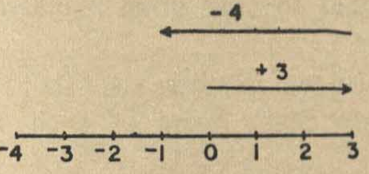

Fig. 2.11

to the left of it. Thus $^{+}3+{}^{-}4={}^{-}1$ (fig. 2.11).

Solve the following problems, using the number-line (fig. 2.12). The point A moves from the point O, first by $^{-}3$ units and then by $^{-}8$ units. What is the position of the point A after these two movements ?

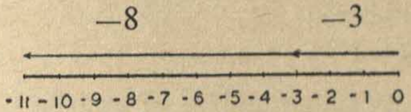

Fig. 2.12

Using the number-line, find the sum of :

(a) $^{-}6$ and $^{-}1$ (b) $^{+}4$ and $^{-}7$ (c) $^{-}6$ and $^{+}3$
(d) $^{+}5$ and $^{-}5$ (e) $^{-}6$ and $^{-}2$ (f) $^{-}9$ and $^{+}12$

46. Find the value of the following by moving on the number-line :

(a) $^{-}140+{}^{+}110$ (c) $^{-}80+{}^{+}46$ (e) $^{-}800+820$
(b) $^{+}240+{}^{-}320$ (d) $^{+}410+90$ (f) $^{+}908+{}^{-}8$

47. Compare :

(a) $^{+}16+{}^{-}6$ and $^{-}16$ (c) $^{-}8+{}^{-}5$ and $^{-}5$
(b) $^{-}20+{}^{-}4$ and $^{-}20$ (d) $^{-}10+{}^{+}1$ and $^{-}1$

48. Check the equalities :

(a) $^{-}52+{}^{-}24={}^{+}40+{}^{-}48$
(b) $^{+}50+{}^{-}16={}^{-}56+{}^{+}16$

49. Are the following inequalities true ?

(a) $(11+{}^{-}9)>({}^{-}10+9)$ (c) $({}^{-}18+{}^{-}5)<({}^{-}18+{}^{-}4)$
(b) $({}^{+}4-{}^{-}24)<({}^{-}3+{}^{-}24)$ (d) $({}^{-}11+{}^{+}6)>({}^{-}2+{}^{-}5)$

50. Find the value of a+b, when :

(a) $a=0,\ b={}^{-}10$ (b) $a={}^{-}7,\ b=0$

51. Write down, in the form of an equality, the most important property of zero. **The sum of a given number and zero is equal to the given number.** Is this equality always true ?

52. Can there be two numbers such that their sum is equal to zero ? How many pairs of such numbers can you think of ?

53. Find the value of the expression $a+b$, if :

    (a) $a={}^{+}20,\ b={}^{-}20$ (b) $a={}^{+}204,\ b={}^{-}204$

Two numbers, whose sum is equal to zero, are called opposite numbers.

54. Write down pairs of opposite numbers from the following row of numbers :

    ${}^{-}17, {}^{+}8, {}^{-}6, {}^{-}4, {}^{-}3, {}^{+}3, {}^{+}6, {}^{+}17, {}^{+}4, {}^{-}8$

55. Find the opposite numbers of each of the following :

    ${}^{-}200, {}^{-}100, {}^{-}65, {}^{-}48, {}^{+}84, {}^{+}95$, 0 and ${}^{-}1$

56. Write each of the numbers ${}^{+}10, {}^{-}7, {}^{+}15$ and ${}^{-}21$ as a sum of two numbers, having the same sign.

57. Write each of the numbers ${}^{+}1, {}^{-}1, {}^{+}2$ and ${}^{-}14$ as a sum of two numbers, with opposite signs.

58. Solve the equations :

    (a) $x+{}^{-}25=0$ (b) $73+a=0$

59. Check, using the number-line, the following equalities :

    (a) ${}^{-}3+{}^{-}6={}^{-}6+{}^{-}3$ (b) ${}^{+}7+{}^{-}2={}^{-}2+{}^{+}7$

60. In the previous exercise we have seen that the commutative law is true for integers.

    Write down this law in a general form (using letters a and b).

61. Check the equalities :

    (a) $({}^{-}6+{}^{-}5)+{}^{-}1={}^{-}6+({}^{-}5+{}^{-}1)$

    (b) $({}^{+}8+{}^{-}3)+{}^{+}5={}^{+}8+({}^{-}3+{}^{-}5)$

In the above exercises we have seen that the most important property of addition is the associative property of addition. This property makes it possible to find the sum of three or more numbers in different ways.

In a general form, the associative law can be written in the following way. For any integers a, b and c,

$$(a+b)+c=a+(b+c)$$

62. When several numbers are to be added, it is useful to add all the positive numbers, all the negative numbers and then add the first sum to the second sum. Thus, simplify the following :

    (a) $^{-}420+^{-}36+^{-}14+^{-}80$

    (b) $^{-}75+^{-}25+^{+}38+^{+}72$

    (c) $^{+}5+^{+}7+^{+}9+^{-}6+^{-}2+^{+}3$

    (d) $^{-}4+^{-}8+^{+}8+^{-}5+^{-}5+^{-}6$

63. Now I am 11 years old. My sister is 'a' years elder to me. How old is she ?

    Explain the meaning of this problem if :

    $a=^{+}18,\ ^{+}3,\ ^{-}2,\ 0,\ ^{-}7$ and $^{+}5$

64. The temperature was 50° in the morning ; it increased by 6° at dinner time but decreased by 2° in the evening. The temperature decreased further by 10° the next morning. What would be the temperature the next morning ? Solve the problem only by adding numbers.

65. Compare the values of expressions $a+5$ and $b+2$, if :

    (a) $a=^{-}2,\ b=^{-}7$ (c) $a=^{+}5,\ b=^{-}2$

    (b) $a=^{-}10,\ b=^{-}8$ (d) $a=^{+}10,\ b=^{-}3$

4. Subtraction

For integers, subtraction has the same meaning as for natural numbers. *To subtract number 'a' from number 'b' is to find number x, such that the sum of x and b is equal to a.*

For example, to subtract number $^{+}5$ from number $^{-}2$ means to find number x, such that the following equality holds :

$$x+^{+}5=^{-}2$$

It is easy to understand that x is equal to $^{-}7$ since $^{-}7+^{+}5=^{-}2$.

66. Subtract :

(a) ${}^{-}3$ from ${}^{+}8$ (b) 9 from ${}^{+}7$ (c) ${}^{-}5$ from ${}^{-}6$
(e) ${}^{+}2$ from ${}^{-}7$

Try to guess unknown numbers

To formulate a rule for solving equations, let us use a number-line. Suppose we are asked to subtract number ${}^{-}8$ from number ${}^{-}5$. It means that number x has to be found which satisfies the equality :

$$x + {}^{-}8 = {}^{-}5$$

To add number ${}^{-}8$ to number x, we find on the number-line number x and moving 8 units to its left come to number ${}^{-}5$. Now it is clear that if we move 8 units from number ${}^{-}5$ to its *right*, we get number x. So, number x is equal to ${}^{+}3$.

But to move 8 units to the right from number ${}^{-}5$ means to add 8 (the number opposite to the subtrahend) to this number.

Hence, we get the general rule for subtraction :

To subtract one number from the other, it is sufficient to add to the minuend a number opposite to the subtrahend.

Thus :

Example 1. $ {}^{-}4 - {}^{-}20 = {}^{-}4 + {}^{+}20 = {}^{+}16$

Example 2. $ {}^{-}5 - {}^{+}10 = {}^{-}5 + {}^{-}10 = {}^{-}15$

Example 3. $2 - 20 = {}^{+}2 + {}^{-}20 = {}^{-}18$

Example 4. $0 - {}^{-}30 = 0 + {}^{+}30 = {}^{+}30$

Rule of subtraction and the above example show that the subtraction of whole numbers can be done even when minuend is less than the subtrahend.

67. Father is 35 years old and uncle is ${}^{+}m$ years old. By how many years is uncle older than father ? If $m = 33$ or 30, explain the meaning of the result. Can m be a negative number ?

68. In the middle of the day the temperature was 17° ; in the evening it dropped by t°. What was the temperature in the evening? Find the value of the expression if $t=5, 0, 2, 7$. Explain the meaning of the results.

69. Make the operations :

(a) $^{+}10+{}^{-}8$ (b) $^{+}12-{}^{-}14$ (c) $^{-}21-{}^{-}19$

(d) $^{+}9-{}^{-}9$ (e) $^{+}20-{}^{+}20$ (f) $^{-}32-{}^{-}48$

(g) $0-{}^{-}25$ (h) $^{-}3-{}^{-}3$

70. Compare :

(a) $^{-}3-{}^{+}12$ and 18 (b) $^{-}2$ and $^{+}3-{}^{-}8$

71. Check the equalities :

(a) $^{-}4-{}^{-}20={}^{-}2+{}^{+}18$ (b) $^{+}4+{}^{+}20={}^{+}2-{}^{-}18$

(c) $7-{}^{-}6={}^{-}12+{}^{+}2$ (d) $^{+}3-{}^{-}25={}^{-}3+{}^{+}31$

72. Find the value of the expression $a-b$, when :

(a) $a={}^{-}10,\ b=3$ (b) $a=0,\ b=12$

(c) $a=b,\ \ b=0$ (d) $a=1,\ b=0$

73. In what case is the difference equal to the minuend ? Write the answer in a general form.

74. In what case is the difference equal to zero ? Write the answer in a general form.

75. Solve the equations :

(a) $x+{}^{-}24={}^{+}18$ (b) $a-{}^{-}52={}^{-}45$

(c) $^{+}76+m={}^{-}5$ (d) $2-x={}^{+}50$

76. Instead of a star, put the sign of equality or inequality :

(a) $^{-}22-{}^{-}32 \ * \ 4+18$ (b) $^{+}5-{}^{-}16 \ * \ {}^{-}6-27$

77. Find the value of the expression $a+b-c$, if :

(a) $a={}^{-}8,\ b={}^{+}5,\ c={}^{-}3$ (b) $a={}^{-}7,\ b={}^{-}4,\ c={}^{+}2$

(c) $a={}^{+}5,\ b={}^{-}5,\ c={}^{-}10$ (d) $a={}^{+}2,\ b={}^{-}20,\ c={}^{-}10$

5. Algebraic Sum

78. Let us agree to notate the number opposite to 'a' as ^{-}a (minus a). So, if $a={}^{-}3$, then $-{}^{-}a=-{}^{-}3={}^{+}3$. Find ^{-}a if : $a={}^{-}8,\ {}^{+}25,\ {}^{-}1,\ {}^{+}68,\ 0$.

Find a, if $^{-}a={}^{+}7,\ {}^{-}15,\ {}^{+}1,\ {}^{-}23,\ {}^{-}36,\ 0$.

79. Find $-({}^{-}a)$, if :

(a) $a={}^{+}4, {}^{-}6, {}^{+}20$ (b) ${}^{-}a={}^{-}2, {}^{+}4, {}^{-}5$

80. Find the value of the expression $b+({}^{-}a)$, if :

(a) $b={}^{-}3,\ a={}^{-}4$ (b) $b={}^{+}2, a={}^{+}5$

(c) $b={}^{+}5,\ a={}^{-}4$ (d) $b={}^{+}7, a={}^{-}2$

81. Find the value of the expression $({}^{-}b)-({}^{-}a)$ if :

(a) $b={}^{-}7,\ a={}^{+}6$ (b) $b={}^{+}8,\ a={}^{-}4$

(c) $b={}^{+}5,\ a={}^{+}7$ (d) $b={}^{-}8,\ a={}^{-}12$

82. Write down the following differences as sums :

(a) ${}^{-}28-{}^{-}32$ (b) ${}^{-}46+{}^{+}30$

(c) $(-x)-a$ (d) $x-m$

83. Represent, as a sum, the following expressions :

(a) ${}^{+}4-{}^{-}5-{}^{+}6+{}^{-}2$ (b) $-7-{}^{-}7-{}^{+}8-{}^{-}4$

(c) $a-({}^{-}b)-c+({}^{-}m)$ (d) $({}^{-}a)+b-({}^{-}c)-(m)$

Find the value of the above expressions if :

$m=3,\ b={}^{+}4,\ a={}^{+}5$ and $c={}^{+}6$

If each of the differences ${}^{+}7-{}^{+}12$ and $a-b$ is expressed as a sum (to be transformed into a sum) we get :

$${}^{+}7-{}^{+}12={}^{-}7+{}^{-}12\ ;\quad a-b=a+(-b)$$

Generally the expressions are written in a shorter way. So ${}^{+}7-{}^{+}12$ can be written as $7-12$. The sign '+' in the beginning of an expression is omitted, *i.e.*, $+a-b$ is simply written as $a-b$. Such expressions are called Algebraic Sums. So, the difference between $a-b$ can be considered as a sum of two numbers a and ${}^{-}b$ or as a difference of the numbers a and b.

84. Write as an algebraic sum the following expressions :

(a) ${}^{+}7-{}^{+}4+{}^{-}5-{}^{-}6$ (b) ${}^{-}68-{}^{-}3-{}^{-}4-{}^{+}1$

(c) $a-({}^{-}b)-({}^{-}c)$ (d) $a+({}^{-}b)-({}^{-}c)$

How many terms does each of the sums have ?

85. Find the values of the algebraic sum :

(i) $-a+b-c-m$ when :

(a) $a={}^{-}3,\ b={}^{-}4,\ c=2,\ m={}^{-}3$

(b) $a=0,\ b=7,\ c=8,\ m=12$

(ii) $a-b-(m+n)$ when :

(a) $a=b=8,\ m=n=8$

(b) $a={}^{-}3,\ b={}^{-}5,\ m=3,\ n=7$

86. Prove that the values of the following expressions are opposite numbers :

(a) $-7+5$ and $7-5$ (b) $-2-8$ and $2+8$

(c) $6+4$ and $-6-4$ (d) $1-20$ and $-1+20$

87. Prove that if a, b, c, m and n are any numbers, the following expressions are opposite numbers :

(a) $a+b$ and $-a-b$

(b) $-a+b$ and $a-b$

(c) $a-b+c$ and $-a+b-c$

(d) $-a-m+n$ and $a+m-n$

In the previous exercises, we have proved that the values of the expressions $-a-b$ and $a+b$ are opposite numbers where a and b are any numbers. But we know that the values of the expressions $-(a+b)$ and $a+b$ are also opposite numbers. Therefore, the equality, $-(a+b)=-a-b$, is true.

Hence to find a number opposite to the algebraic sum, it is sufficient to change the signs of each term.

We also know that $+(a+b)=a+b$.

Now we can formulate the most important rule of transformation of expressions. When removing the brackets, (1) *the signs of the terms are not changed if the sign '+' precedes the brackets* ; (2) *the signs of the terms are changed if the sign '–' precedes the brackets.*

Example 1. Find the value of the expression :

$$-6+4-(-6+4-8)$$

The expression within brackets is preceded by the sign '–'. Therefore, to remove the brackets, change the sign of each term.

$$-6+4-(-6+4-8)=-6+4+6-4+8=8$$

Example 2. In $a-b+c-(a-b+m-k)$; the sign '–' precedes the brackets. So to remove the brackets, change the sign of each term.

$$a-b+c-(a-b+m-k)=a-b+c-a+b-m+k=c-m+k$$

Example 3. Transform the expression $a+b-c+(m-b+c+k)$.

If the sign '+' precedes the brackets, then, for removing the brackets, do not change the signs of the terms.

Thus

$$a+b-c+(m-b+c+k)=a+b-c+m-b+c+k$$
$$=a+m-k+(c-c)+(b-b)=a+m-k$$

88. Find the value of the following expression :

$-a-k-(-a+b-m-k)$ if $a=-6$, $b=-8$ $m=5$, $k=3$,

89. Remove the brackets :

(a) $-(-a-b-c-d)$ (b) $-x-(m-a+b)$

(c) $-a-(m-a-d)$ (d) $m+(k-a-m)$

90. Enclose within brackets the last two terms putting the sign '+' before the brackets :

(a) $-3+4+7-5$ (b) $-m+a-n-b$

91. Enclose within brackets the last three terms of each expression putting the sign '−' before the brackets :

(a) $-6-2+4-3$ (c) $-a+b+c+d$

(b) $a-b-c-d$ (d) $m-n-k-a+x$

92. Find the value of the expression $-(m+n)$ and $-(m-n)$, when :

(a) $m=8$, $n=2$ (b) $m=9$, $n={}^{-}4$

93. Write as an algebraic sum the following expressions :

(a) $8+(-25)-(-60)$ (b) $-1-(-2)+(-4)$

(c) $-a+(-b)-(-m)$ (d) $a-(-b)+(-c)$

94. Find the value of the expression :

$-c-(-a)-(-b)$, when :

(a) $x={}^{-}3$, $a=4$, $b=5$ (b) $x=5$, $a={}^{-}7$, $b={}^{-}3$

95. Prove that $a-b$ and $b-a$ are opposite numbers, when a and b are any two integers.

96. Enclose within brackets the first three terms in each expression putting the sign '–' before the brackets :

(a) $-6+8-7+3-4$ (b) $a+b-c+m$

97. Find the numerical value of the expression

$a-(b-m)-m$, when :

(a) $a={}^{-}8,\ b={}^{-}4,\ m={}^{-}5$

(b) $a=6,\ b={}^{-}7,\ m=8$

6. Multiplication

We know the meaning of the multiplication of natural number, *viz.*,

$7\times 4=7+7+7+7=28$

The same meaning may be given to the multiplication of a negative number by a positive number, *e.g.*,

$$-8\times 5=(-8)+(-8)+(-8)+(-8)+(-8)=-8-8-8-8-8$$
$$=-(8+8+8+8+8)=-(8\times 5)=-40$$

So, $-8\times 5=-(8\times 5)$

We get the rule for the multiplication of a negative number by a positive number : *the negative multiplier is to be replaced by the opposite number and after finding the product put the minus sign before the product.*

The same rule is used for the multiplication of a positive number by a negative number, *i.e.*,

$8\times(-5)=-(8\times 5)=-40$

98. Carry out operations :

(a) $(-4)\times 5$ (b) $8\times(-9)$ (c) -40×30

(d) $12\times(-14)$

99. Find the value of the expression $7a+4b$ if :

(a) $a=-6,\ b=-4$ (b) $a=4,\ b=-6$

Product of two negative numbers can be considered as equal to the product of the numbers opposite to these numbers.

Example : $-7\times(-6)=7\times 6=42$

If one of the factors is equal to zero, the product is equal to zero.

100. Solve:

(a) $-5\times(-6)$ (c) $-25\times(-4)$

(b) $-8\times(-20)$ (d) $-125\times(-3)$

101. Find the value of the expression $-5a-7x-8m$, if:

(a) $a=4$, $x=-2$, $m=-3$

(b) $a=-3$, $x=4$, $m=5$

102. Express each of the numbers 8, -22, -41, 32 and 7 as the product of two numbers such that atleast one of them is a negative number.

103. The commutative law of multiplication is true for integers also. Using letters a and b, write it in a general form. Prove that it is true when:

(a) $a=-4$, $b=5$ (c) $a=-3$, $b=-8$

(b) $a=6$, $b=-2$ (d) $a=0$, $b=-7$

104. The associative law is also true for multiplication of integers. This law can be written in this way:

$(ab)\times c=a\times(bc)$

Check this equality, when:

(a) $a=3$, $b=-2$, $c=4$ (b) $a=-5$, $b=-4$, $c=-3$

105. Multiply:

(a) $(-2)\times36\times(-5)$ (b) $-8\times(-43)\times0\times(-37)$

106. Compare abc and 0 when:

(a) $a>0$, $b>0$, $c>0$ (c) $a<0$, $b<0$, $c<0$

(b) $a>0$, $b>0$, $c<0$ (d) $a<0$, $b<0$, $c=0$

107. What number, positive or negative, do we get on multiplying numbers of the type:

(a) 1 negative and 3 positive

(b) 2 negative and 5 positive

(c) 8 negative and one positive

(d) 21 negative and one positive

To find the product of several multipliers it is sufficient:

(1) *to replace all the negative factors by the opposite numbers.*

(2) *to carry out multiplicatian.*

(3) *if the number of negative factors is odd, put minus sign before the product ; otherwise plus sign.*

Example 1. $-7\times(-2)\times 8\times -4)\times 5=-(7\times 2\times 8\times 4\times 5)$
$=-2240$

Example 2. $-3\times(-2)\times(-2)\times(-2)\times(4\times 5=3\times 2\times 2\times 2\times$
$5\times 4=480$

108. Find the values of :

(a) $-125\times(-8)\times 35\times(-42)$ (b) $-2\times(-3)\times(-4)\times$
$(-5)\times(-6)$

109. Express as a sum of :

(a) $-a-a-a-a$
(b) $-b-b.........-b$ to n terms
(c) $-8-8-8.........$to x terms
(d) $-2a-2a...........-2a$ to 10 terms

110. Express the expressions $-3a$, $-ab$ and $-a$ as prodnct of two factors, one of which is the number $-a$

111. Check the equality $(a+b)\,c=ab+bc$ when

(a) $a=-3,\ b=2,\ c=-4$
(b) $a=-1,\ b=-2,\ c=-3$

This equality expresses the distributive law of multiplication.

By this law we can remove the brackets in the product of two factors, one of which is an algebraic sum.

Example 1. $a(b-c)=ab-ac$

Example 2. $-m(a+b-c)=-ma-mb+mc$

112. Using the distributive law, remove the brackets :

(a) $-5(-a+b-c)$ (c) $-2a(b+2c-3m)$
(b) $-6(ab-m+n)$ (d) $4m(-2a+3b-5c)$

113. Using the distributive law, write down the following sums as products :

(a) $am-an-ab$ (d) $-abc+abm-acm$
(b) $-3m+3a-3b$ (e) $-12ma-8mb-16mc$
(c) $tam-tan+tap$ (f) $-24ax-18bx-3c0x$

Note that for finding the numerical value of the algebraic sum $-3ab+3am-3ac$, it is advisible to express it as a product like $-3a(b-m+c)$.

When an algebraic expression is the product of two or more expressions, each of these expressions is called a factor of the given expression.

114. Resolve into factors :

(a) $ab-bb-cb$ (d) $kx-5x-ax$

(b) $-bcx-abx-cxd$ (e) $-10ab-15ac-20ab$

(c) $amb+anb+anb$ (f) $36a-10a-13a$

Terms in the expression $-6a+5a-7a-3a+a$ differ from each other only in their numerical factors. Such terms are called *Like Terms.*

Take the common factor a and make the addition within brackets :

$$-6a+5a-7a-3a+a=a(-6+5-7-3+1)$$
$$=a(-10)=-10a$$

115. Simplify :

(a) $-18n-12n+5n-n$

(b) $-60kx+kx-30kx+100kx$

(c) $4nx-6nx-nx-7nx$

116. Represent each of the numbers -40, 16, -54, -68 and 0 as a product of two factors if one of them is -1.

117. Make the operations :

(a) $-6(-5)\times(-50)\times2$

(b) $-2\times(-3)\times(-4)$

(c) $-2\times(-3)\times(-4)\times(-5)\times(-6)$

118. Compare $a\times b$ and 0 when :

(a) $a>0,\ b>0$ (b) $a<0,\ b<0$

119. Compare x and 20 when :

(a) $x>0$ (b) $x=0$ (c) $x<0$

120. Remove brackets :

(a) $-8(-3a+2b-8c)$ (b) $-m(-5a-4x-6b)$

(c) $-6a(2m-4n-k)$ (d) $5(-3a+4m+8)$

121. Simplify :

(a) $31b-12b+b-b$ (b) $-10mx-9mx+mx-7mx$

7. Division

As for natural numbers, division is defined as an operation inverse of multiplication for integers. To divide an integer a by an integer b is to find an integer x, which when multiplied by b gives a, *i.e.*, if $a \div b = x$ then $a = bx$.

For example, to divide -8 by 4 means to find a number x, such that :

$$x \times 4 = -8$$

We can guess that x is equal to -2 which is true as $-2 \times 4 = -8$. For division we use different ways of notation, for example,

$$-8 \div 4 = -2 \text{ or } \frac{-8}{4} = -2$$

122. Find the quotient by writing the equation and guess the unknown number :

(a) $36 \div (-9)$ (c) $-48 \div (-16)$

(b) $-24 \div 8$ (d) $0 \div (-7)$

123. What number (positive or negative) will be the quotient when :

(a) the dividend and divisor have same signs,

(b) the dividend and divisor have different signs ?

To find the quotient, it is sufficient :

(1) *to replace negative numbers by their opposite numbers,*

(2) *to make the division as in natural numbers,*

(3) *to put minus sign if the dividend and divisor have different signs and plus sign if they have the same sign.*

Example 1. $-56 \div (-4) = 56 \div 4 = 14$

Example 2. $-60 \div 12 = -(60 \div 12) = -5$

Example 3. $42 \div (-7) = -(42 \div 7) = -6$

Now suppose we are asked to divide the number -4 (which is not equal to zero) by zero. If the number a is a quotient then multiplying it by the divisor 0, we should get the dividend -4. But the product $a \times 0$ is equal to 0, and not -4. Therefore we conclude **that the division by 0 is impossible.**

124. Read the left and right sides of the following equalities ;

(a) $2x+7=-15$ (c) $ax-5a=0$

(b) $ab-bc=(a-c)b$ (d) $\frac{a}{x}+1=-6+b$

125. Is the equality a $(1+b)$ true when :

(a) $a=-5$, $b=-1$, (c) $a=2$, $b=-2$,

(b) $a=-1$, $b=-5$, (d) $a=-2$, $b=2$?

126. Given that $a=b$, compare :

(a) $a+3$ and $b+3$ (c) $a+0$ and $b+0$

(b) $a-7$ and $b-7$ (d) $a-0$ and $b-0$

We have seen the very important property of an equality, *i.e.*, if $a=b$, then $a+m=b+m$ where a, b and m are any numbers. Equalities $a=b$ and $a+m=b+m$ are very closely connected with each other. If the first of them is true, then the second will also be true. In such cases it is said that the second equality follows from the first one. In short notation, we write it as, if $a=b$, then $a+m=b+m$, which is read in the following way : From the equality $a=b$ follows the equality $a+m=b+m$.

Read the sentence : $a=b$, $a-m=b-m$ and explain its meanings.

127. What do the following sentences mean ?

(a) $a+m=b+m$ if $a=b$

(b) $a-m=b-m$ if $a=b$

Are these true for any values of a, b and m ? Check your answer when $a=-6$, $b=3$ and $m=-4$.

128. Which of the following expressions are correct ?

(a) $(a-b)=0$ if $a=b$

(b) $(a-b)>0$ if $a<b$

(c) $(a-b)<0$ if $a>b$

(d) $a=b>0$ if $a>0$

(e) if $a>b$, $b>c$ then $a>c$

(f) if $a<b$, $a>c$ then $b>c$

(g) if $a>b$, then $(a+m)>(b+m)$ where $m>0$

(h) if $a<b$, then $(a-m)<(b-m)$ where $m>0$

129. Given $a=b$, compare :

(a) $a\times(-4)$ and $b\times(-4)$ (c) $a\times 0$ and $b\times 0$

(b) $a\times 5$ and $b\times 5$ (d) am and bm

130. If $a=b$, then $am=bm$. Explain its meaning and check if it is true when :

$a=b=4$, $m=5$

131. Given $a=b$, compare :

(a) $\frac{a}{-5}$ and $\frac{b}{-5}$, when a is divisible by -5

(b) $\frac{a}{m}$ and $\frac{b}{m}$, if a is divisible by m

132. Read the sentence if $a=b$ and if b is divisible by m, then $\frac{a}{m}=\frac{b}{m}$. Explain its meaning and check it when :

$a=b=-60$, $m=-12$

133. Which of the following expressions are true ?

(a) $a=b$ then $a(-3)=b(-3)$

(b) $a=b$ then $7a>7b$

(c) $a>b$ then $-5a<-5b$

(d) $a<b$ then $2>2b$

We have found the following four important properties of equalities :

(i) if $a=b$ then $a+m=b+m$

(ii) if $a=b$ then $a-m=b-m$

(iii) if $a=b$, then $am=bm$

(iv) if $a=b$ and a is divisible by m then $\frac{a}{m}=\frac{b}{m}$

These properties can be stated in words as below :

If equal or the same quantities are added to or subtracted from equal quantities, the sum or the difference is equal.

If equal quantities are multiplied or divided by the same or equal quantities (for division, this quantity should not be equal to zero) the results are equal.

Properties of equalities are used in solving equations.

Example 1. Solve the equation $-x-7=-4$. To both sides of the equality (equation) we can add the number 7.

$$-x-7+7=-4+7$$
$$-x=3$$
So, $x=-3$

Example 2. Solve the equation $8-x=-12$. From both sides of the equation subtract the number 8.

$$8-x-8=-12-8$$
$$-x=-20$$
Hence $x=20$

Example 3. Solve the equation $-3x+7=-23$. From both sides of the equation subtract 7.

$\therefore\ -3x+7-7=-23-7$

or $-3x=-30$

Divide both sides of the new equation by -3.

$\therefore\ \dfrac{-3x}{-3}=\dfrac{-30}{-3}$ or $x=10$

Check $-3x+7=-23$ for $x=10$

Example 4. Solve the equation $-3x-18=-5x-12$. Add $18+5x$ to both sides of the equation

$\therefore\ -3x+18+5x-18=-5x+5x+18-12$

or $2x=+6$

Divide both sides of the new equation by 2.

So, $x=+3$

Check your result.

134. Solve the following equations using the properties of equalities :

(i) $-2-x=x-18$

(ii) $6a-1=-10+3a$

(iii) $-3(2x+1)=-33-3x$

(iv) $4(7-5m)=-20+4m$

(v) $-40(-7x+5)=-1600$

(vi) $(-20x-50)\times 2=100$

(vii) $21(4-6a)=-420$

(viii) $-3=[(2-x)-1]$

135. In three days 830 kg of oranges were sold. On the second day, the sale was 30 kg less than that of the first day, and on the third day the sale was 2 times that of the second day. How many kilogrammes of oranges were sold on the first day ?

136. During my vacations I covered by train a distance of 120 km more than that by ship. If I had covered by the train a distance 4 times and by the ship 8 times of the previous distances, the total distance covered would have been 1200 km. How many kilometres did I cover by the ship ?

137. There are 18 litres of water in a jug and 26 litres in a cistern. Some quantity of water is poured from the cistern into the jug such that the quantity of water in the jug and cistern becomes the same. How much water is now in the jug ?

138. There are some hens and rabbits in a cage. Total number of heads is 19 and that of legs is 62. How many hens and rabbits are there in the cage ?

139. Ram's step is 12 cm longer than Mohan's step. But Ram's 4 steps are shorter than Mohan's 6 steps by 54 cm. Find the length of the step of each boy.

140. Solve the following equations :

(i) $-16(8-9x)=592$

(ii) $24\ (5+11x)=-1200$

(iii) $8(-8x+6)=-16$

(iv) $-6(5-11x)=36$

141. A number is 7 times the other. Find these numbers if their difference is equal to 516.

142. A number is greater than the other number by 609. Find these numbers if their sum is equal to 1219.

143. The perimeter of a triangle ABC is equal to 94 cm. AB is 15 cm shorter than BC and AC is 22 cm longer than AB. Find the length of the side BC. What other elements of the triangle ABC can you find ?

144. 26 books and some magazines cost Rs. 128.40 p. How many magazines are there if a magazine costs 60 p and a book Re. 1·40 p.

145. Is $(7x-5x)\,x=800$ true ;
when $x=20$? Is the equality true when $x=0$?

146. Is the statement $(26-2a)\times(a-3)=8$ true when $a=5, 3$?

147. The length of a rectangular field is 3 times its width. The width is 34 metres less than the length. Find the area of the field.

148. The speed of a plane is 16 times the speed of a car. Find the distance covered by the car in 6 hours if its speed is 750 km less than the speed of the plane.

149. If each of the two given numbers is multiplied by 5, the sum of the products will be equal to 500. If the greater number is increased by 16, their difference will be equal to the greater number. Find these numbers.

150. It is known that 7 years ago father was 7 times older than his son. How old is the son, if his father is 49 years old ?

151. Starting from 3, write down such a row of numbers in which each term is less by 2 than the preceding term.

Does the number -722 belong to this row ? Has this row an end ?

152. Find the rule of forming the following row of numbers : $-5, -10, -15, -20, -25, \ldots\ldots$ Find the number on the 4th place and on the 10th place.

153. Find the rule of formation of the row : $2, -4, 8, -16, 32, -64, \ldots\ldots$

154. Check the equality $2a\times a-(a+a)\times a=0$ when $a=10$. Is the eqality true, when $a=0$? Check the equality $(7-x)\times(x-7)=1$ when $x=1$.

155. Find the value of the expression $3a+4b-5c$ when : $a=b=c=505$.

156. Find the value of the expression $(ab-bc)\times 308$, when $a=c=40$. Is there any extra information ? If so, give it.

157. The length of a rectangle is 30 cm and it is 4 times its width. Find the perimeter of the rectangle.

158. If each of the two given numbers is decreased by 4, their difference will be equal to 63. If the smaller of them is decreased by 88, then their sum will be equal to the greater number. Find these numbers.

159. In 7 years father will be 3 times older than his son. Now father is 47 years old. How old is the son ?

160. Starting from the number -1, write down the number row in which each term is 3 less than the preceding one. Is there the number —49 in this row ?

161. Find the rule of formation of the row, 2, —4, 6, —8, 10, ... 18....... Will you have in this row the numbers —26, 38, —38, 47 and —61 ? What number is in the 4th and the 14th place ?

162. A row was written in which each term, beginning from the third, is equal to the sum of two preceding terms : —89, 55,......

Are there two neighbouring (adjacent) terms with the same sign ? Compare the second and the 22nd terms.

CHAPTER III

FRACTIONS

1. Common Fractions

If a circle is divided into 5 equal parts, then each of these parts is called one-fifth of the circle. If we cut a wooden board into 6 equal parts, we shall call each of these parts as one-sixth of the board.

If we divide some quantity into 40 equal parts, we shall call each part as the fortieth part of it. These can be written as : 1/5 of a circle, 1/6 of a board, 1/40 of a quantity.

1. What is the name for :

   (a) one-tenth of a tonne, (b) one-hundredth of a quintal
   (c) one-sixth of a minute, (d) one-twentyfourth of a day ?

2. In each of the following figures point out the shaded part.

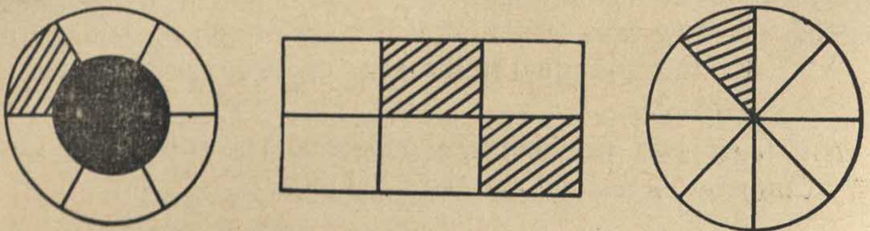

Fig. 3.1

3. What part of the segment CD (fig. 3.2) is the segment MN ? What part of the segment AB is the segment CD

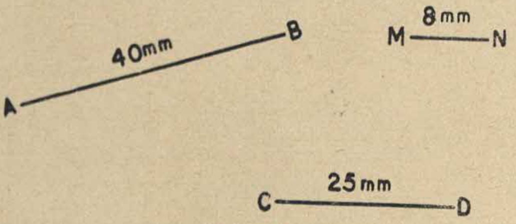

Fig. 3.2

4. What part of a metre is a decimetre ; a centimetre and a millimetre ?

5. What part of a tonne is a quintal ; a kilogramme ; a gramme ?

6. What part of a square metre is a square decimetre ; a square centimetre ?

7. What part of an hour is a minute and a second ?

8. What part of a straight angle is a right angle ?

9. How many one-fourths are there in a half of a segment ?

10. How many eight parts of a segment are there in its half ?

11. How many hundredths of a metre are there in its one-tenth part ?

12. Cut out a circle from a piece of paper. Cut it into four congruent parts.

13. Can you cut a square into four congruent parts without cutting its sides ?

14. Length of the sides of a rectangle are 6 cm and 4 cm. Draw (a) its $\frac{1}{3}$; (b) its $\frac{1}{4}$.

15. The segment shown in fig 3.3 represents an hour.

    Draw a segment representing :

    (a) 1/2 of an hour

    (b) 1/3 of an hour

    (c) 1/4 of an hour

    (d) 1/6 of an hour

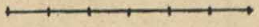

Fig. 3.3

6. What part of a day is an hour and a day of a month ?

17. How many fifteenths of a segment will make :

    (a) one-fifth, (b) one-third of it ?

A circle is divided into 8 equal parts and two of them are shaded in fig. 3.4. We can write it as 2/8 of a circle.

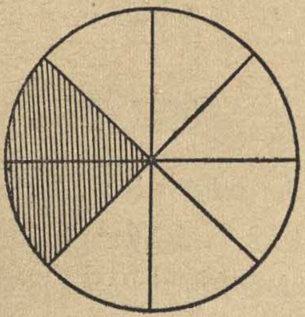

Fig. 3.4

18. A rectangle is divided into 5 equal parts and then another rectangle, containing 6 of these parts, is drawn (fig. 3.5). This new rectangle is 6/5 of the first rectangle.

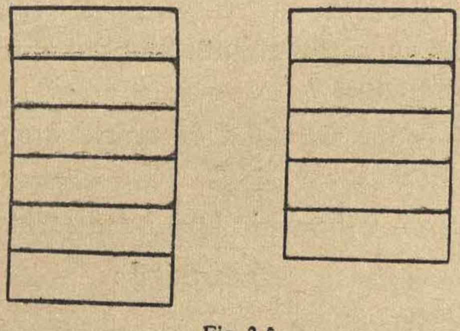

Fig. 3.5

19. Draw a circle and a rectangle. Shade 3/4 of each figure.

20. What part of each of the following figures (fig. 3.6) is :
(a) shaded, (b) unshaded ?

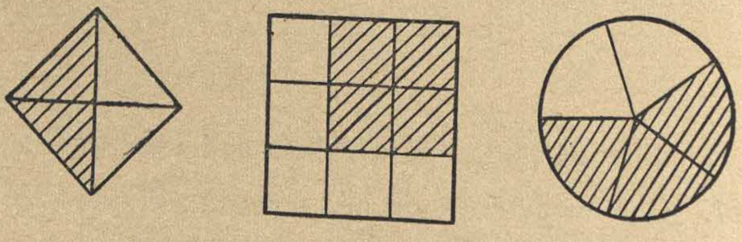

Fig. 3.6

21. A lamp can burn for 5 hours with one litre of kerosene. What part of a litre of kerosene will be burnt in :

(a) 1 hour, (b) 3 hours, (c) 4 hours ?

22. Find by measuring what part of the segment AB is :

(a) the segment CD,

(b) the segment EF,

(c) the segment OP ?

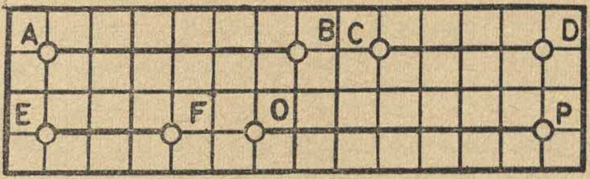

Fig. 3.7

23. Explain the meaning of each of the following sentences :

(a) 5/8 of a cake has been eaten

(b) 4/5 of a distance has been covered

(c) 2/3 of a vessel has been filled

24. If a segment, whose length is equal to 12 cm represents one kilometre, draw segments to represent :

(a) 2/3 km (b) 4/3 km

25. A plane covers a distance between two towns, A and B in 6 hours. What part of the distance does it cover in 4 hours and 5 hours ? Make a drawing.

2. Representing Fractions on a Number-line

We know such numbers as 1/2 ; 1/10 ; 2/6; 7/4 ; 3/3. All these numbers are called fractions. Let us consider the following examples : 2/5 of a circle ; 2/5 of a rectangle ; 2/5 of a segment. In each of these examples the number 2/5 shows that a certain given part or quantity (or object) is divided into 5 equal parts and two of these have been taken. The number 5 is called the *denominator* of the fraction, and the number 2 is called the *numerator* of the fraction (fig. 3.8).

You can express any natural number and also the number 0

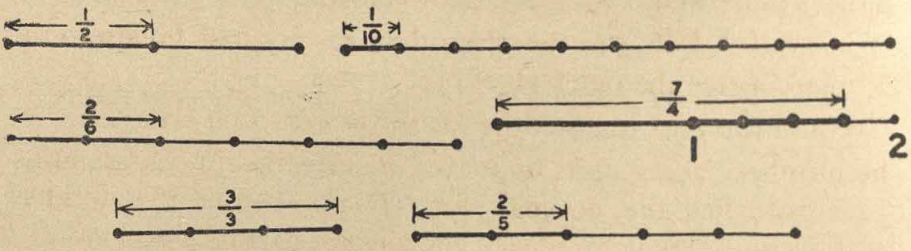

Fig. 3.8

(if it is necessary) as fraction, with denominator 1. For example :

7=7/1, 25=25/1, 0=0/1

For any fraction, as is done in the case of natural numbers, an opposite fraction is introduced, for example :

+1/2, −1/2 ; +3/1, −3/1 ; −6/4, +6/4 ; −8/8, +8/8

26. Name the numberator and the denominator of each of the fractions :
 8/2, 8/12, 7/1, 1/8, 25/4 and 3/40
27. Draw a numer-line. Mark on it the following numbers :
 2, 4, —1, −2/3, −7/3.
 (scale : 1 unit=3)
28. Explain the meaning of the following sentence :
 'A train leaves after t hours',
 when t =3/4, −1/2, −3/4, 0
29. Mark the following numbers on a number-line :
 −2, −7/6, −1/6, 1/6, 4/6, −4/6
 (scale : 1 unit=6)
30. Write down as directed :
 (a) 5 dm 3 cm and 4 dm 9 cm in decimetres
 (b) 14 m 2 dm and 5 dm in metres

3. Reading and Writing Decimals

Suppose, that while measuring a segment by metres we have got not only its tenth (decimetres) but its hundredth (centimetres) also. For example, 5 m, 4 dm 6 cm= (5 'ones', 4 'tenths' and 6 'hundredth' m)=5·46 m.

So, each digit in the number 5·46 represents a certain value according to its place in the numeral. The digit 5 is in unit's place, 4 is in tenth's place and 6 is in hundredth's place.

The number 5·46 has been written in decimal notation, the **Decimal point** locates the one's place, and is read :

'Five and forty six hundredths' or 'Five point four six'.

The number having units of tenths, hundredths or thousandths and so on is called the **decimal part.** Digits before the decimal point form a number which is called the **integral part** of a decimal.

Example : Express 27 p in rupees. Remembering that a paisa is one hundredth part of a rupee, we shall have :

27 p = Re. 0·27

Example : Express 6 t 38 kg in tonnes.

In 6 t 38 kg there are 6 tonnes and 38 its thousandths (a kilogramme is one-thousandth part of a tonne), so :

6 t 38 kg = 6·038 t

31. Read the numbers :

    (a) 387·8 (c) 6555·5 (e) 8000·1
    (b) 2408·6 (d) 4007·9 (f) 27000·3

32. Tell the integral and fractional parts of the following numbers :

    (a) 8·4 (b) 23·6 (c) 0·7 (d) 5708·1

33. Express in millimetres :

    (a) 8·9 cm (b) 26·3 cm

34. Mark on a number-line the following numbers :
    $0{\cdot}7, -0{\cdot}5, 0{\cdot}8, 1{\cdot}2, -0{\cdot}3$ (scale : 1 unit = 10)

35. Express as desired :

    (a) 807 m 4 dm and 0·8 dm into metres
    (b) 9 cm 2 mm and 9 mm into centimetres

36, Express :

    (a) 21·4 dm and 0·8 dm into centimetres
    (b) Rs. 2·40 and Re. 0·03 into paise

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37. Using the place value chart given below, read the number 98403·2765.

Place Value Chart

tens of thousands	thousands	hundreds	tens	units	decimal pt	tenths	hundredths	thousandths	ten thousandths
9	8	4	0	3	.	2	7	6	5
9	8	4	0	3	.	2	7	6	5

38. Read the following decimals :

(a) 4·56, 1·40, 0·05 (b) 6·1678, 0·0345, 0·2186

39. Express as directed :

(i) $\left.\begin{array}{l}\text{(a) 7m 5 dm 5 cm}\\ \text{(b) 24·3 dm 6 cm}\end{array}\right]$ in metres

(ii) $\left.\begin{array}{l}\text{(a) 68 dm 7 cm 5 cm}\\ \text{(b) 9 cm 1 mm}\end{array}\right]$ in decimetres

(iii) (a) Rs. 2 and 9p (b) 3p (c) 12p into rupees

(iv) $\left.\begin{array}{l}\text{(a) 5 t 8 kg}\\ \text{(b) 9 kg}\end{array}\right]$ in tonnes

(v) $\left.\begin{array}{l}\text{(a) 1 km 75 m}\\ \text{(b) 3 km 9 m}\end{array}\right]$ in kilometres

40. Write down in decimal fractional form :

(a) twenty five and seven hundredths
(b) fifty five and seventy hundredths
(c) sixty four thousandths
(d) eighty eight and five thousandths

4. Comparison of Decimals

Equality of Decimals

Mark on a number-line the numbers 2 and 2·0. You see that they are represented by the same point. Two numbers are considered as equal if they are represented by the same point on a number-line. If you mark on the line, numbers 3·4 and 3·40 you will see that these numbers are also equal. This condition of equality has practical value. For example, if you measure the length of the black-board in the classroom and get the result 3m 4 dm, that is, 3·4m. But the length of the same black-board is 3 m 40 cm, that is equal to 3·40 m. Decimals 3·4 and 3·40 express the same length, therefore they are equal, *i.e.*, 3·40 = 3·4 or 3·4 = 3·40

You can get equal decimals on the basis of the following rule : *The value of a decimal will not change if a zero is added to the right of it. If a decimal ends by a zero, the zero can be omitted.* This rule can be used when after cancelling the zero, a whole number is left, for example, 8·0 is equal to 8. This rule enables us to write any whole number as a decimal as 25=25·0 ; −32=−32·0 and 6=6·0.

41. Express in metres :

    (a) 27 m 40 cm (b) 65 m 90 cm

42. (i) Express in tonnes :

    (a) 4 t 750 kg (b) 400 kg

    (ii) Express in quintals and kilogrammes :

    (a) 7·80 q (b) 5·4 q

43. Measure the width of a classroom and express the result in metres.

44. To each of the decimals 0·5, 6·42, −2·8 and −76·65 add two zeros at the end and read the numbers which you get.

45. Take a thermometre and read the temperature from it.

Inequality of Decimals

Now we shall extend the meanings of the words 'less than', and 'greater than' to decimals. Out of two decimals the lesser decimal is that which is to the left of the other decimal on the number-line.

Mark on a number-line the numbers 1·3, 0·9, −2·1 and −1·8. Then you will see that the following inequalities are true :

1·3>0·9 ; −2·1<−1·8 ; 0·9>−·8

It is easy to compare decimals without graphing them on a number-line. For example, two decimals 2·8 and 2·789 are to be compared. We know that the number 2·8 is equal to the number 2·800. The whole parts of the decimals are equal. But in the first decimal there are 800 thousandths and in the second, 789. So, the first number is greater than the second one.

To compare decimals, **we compare their whole parts. If they are equal, then we compare the digits on the tenth's and hundredth's place and so on.**

46. Mark on a number-line the numbers :
−1·4, −1, −0·8, −0·5, 0·4, and compare :
(a) −1·4 and −1 (c) 0 and −1·2
(b) −0·5 and −0·8 (d) −0·5 and 0·4

47. On the number-line given below a row of numbers is marked. Between what successive integers are the following numbers :

(a) 0·8	(d) −3·7	(g) 0·435
(b) 2·9	(e) 1·65	(h) −2·45
(c) −1·3	(f) 3·807	(i) 0·349

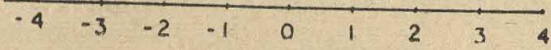

Fig. 3.9

48. On the number-line given below a row of numbers is marked. Between what successive marked numbers do the following numbers lie ?
(For example, −2·15 is between −2·1 and −2·2)
(a) −1·74 (c) −2·169
(b) −2·03 (d) −1·834

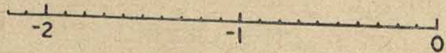

Fig. 3.10

49. Put the sign < or > instead of the stars :
(a) 2·1 * 1·876 (c) −7 * −0·608
(b) 8·006 * 9·5 (d) −4·2 * 5·8

50. Place in the ascending order the numbers :
1·25, 0, −0·3, 5·621, −8·25, 0·13

51. Place in the descending order the numbers :
6·125, −7·3, −5, 0, 14·1, −7·6

52. What integers are there between the following decimals ?
(a) 2·9 and 4·2 (c) −9·1 and −8·71
(b) −5·6 and −7·9 (d) −1·2 and 1·2

53. What digit should be put instead of the star in 2·*1 > 2·81 to make the inequality true ?

54. What digit should be put instead of the star 0·7*5 < 0·725 to make the inequality false ?

55. Show a row of numbers of your choice on a number-line. Between what successive terms of the row are the numbers ?
(a) 0·6 (b) −2·9 (c) −1·4 (d) −3·4

56. Put the sign < or > instead of the stars :
(a) 3·018 * 8·5 (c) −8 * 0·452
(b) −2·6 * −6·2 (d) −7·45 * −4·52

57. Between what successive integers are the following decimals ?
(a) −9·9 (b) 0·754

58. Write down all the possible inequalities using the following numbers :
−4·5, −8·52, 0·42

5. Addition and Subtraction

Addition and subtraction of positive decimals

Problem : 1·45 metres of woollen cloth was used for a coat and 2·8 metres for a suit. What was the total quantity of cloth used ?

Calculate the total quantity of the cloth. The whole number of metres is 3; decimetres (tenths of a metre) are 12, that is, 1 m 2 dm and centimetres (hundredths of a metre) are 5. The total quantity is 4 m 2 dm 5 cm or 4·25 m. The number 4·25 is called the sum of the numbers 1·45 and 2·8.

When you add by writing numbers, it is better to start with the units of the smallest value (as you have done when whole numbers were added) and write the given numbers in a column, placing the decimal points directly under one another (see example 1).

Subtraction of decimals has the same meaning as the subtraction of integers. To subtract one number from another means to find a third number, which, when added to the second, gives the first one.

Suppose 5·6 is to be subtracted from 8·4. It means that a number x is to be found, such that the sum of 5·6 and x should be equal to 8·4. The number x is equal to 2·8 as 5·6+2·8=8·4. Usually it is written as 8·4−5·6=2·8.

To subtract one number from the other, write these numbers one below the other. Decimal points are written under one another (see example 2).

Example 1 :

49·68+256·389=306·069

$$\begin{array}{r} 49{\cdot}680 \\ +256{\cdot}389 \\ \hline 306{\cdot}069 \end{array}$$

Example 2 :

58·3−6·384=51·916

$$\begin{array}{r} 58{\cdot}300 \\ -6{\cdot}384 \\ \hline 51{\cdot}916 \end{array}$$

59. 4·7 m were cut off from a piece of rope. How many metres were left if there were 30 metres of the rope in the beginning ?

60. A book is cheaper than an album by Re. 0·79. How much does the album cost if the book costs Re. 0·99 ?

61. A man covered 15 km in an hour on a bicycle. If he had walked he would have covered 9·7 km less. Find his rate of walking.

62. Find the values of the expressions :

(a) 0·2727+1253·4 (c) 96·3+0·081

(b) 9·1 −5·46 (d) 1 −0·999

63. Represent each of the decimals 7·365, 8·7 and 0·45 as the sum of its whole part and decimal fraction (less than 1).

64. Represent each of the decimals 6·59, 27·8 and 0·381 as the difference of a natural number and a decimal fraction (which is less than 1).

65. One of the sides of a triangle is 83·6 cm, the second side is 23·4 cm longer than the first, and the third side is 8·6 cm shorter than the second. Find the perimeter of the triangle.

66. Find the value of the expression m−n, if :

(a) m=1, n=0·768

(b) m=32·5, n=1·567

Addition and Subtraction of any Decimal

The rule of addition of whole numbers on the number-line is extended to the addition of any decimals. Suppose the number 8·6 is to be added to the number−7·3. Find the number −7·3 on the number-line (figure 3.11) and on placing to the right of this number a segment, whose length is 8·6 units, we will find the sum of the numbers −7·3 and 8·6. Hence −7·3+8·6=1·3.

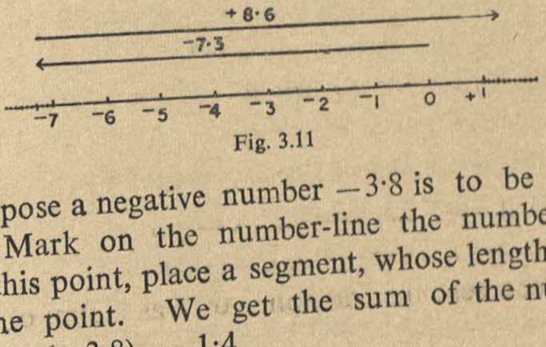

Fig. 3.11

Now suppose a negative number −3·8 is to be added to the number 2·4. Mark on the number-line the number 2·4. Then starting from this point, place a segment, whose length is 3·8 units to the left of the point. We get the sum of the numbers 2·4 and −3·8, *i.e.*, 2·4 + (−3·8)=−1·4.

Adding zero to any number does not change the number. So for any value of *a* there is the equality :

$$a + 0 = a.$$

67. Are the commutative and the associative laws true for the addition of decimals ?

Subtraction of decimals has the same meaning as the subtraction of integers and positive decimals. To subtract a number b from another number a means to find a number x, which when added to the number b gives the number a.

Rules true for the subtraction of integers apply to the subtraction of decimals also. Therefore, the following rule is true for subtraction of decimals : *To subtract one number from another, it is sufficient to add to the minuend the number opposite to the subtrahend.*

In addition and subtraction of decimals the rules of removing and enclosing in brackets can be used.

68. Remove the brackets :

(a) $3{\cdot}6-(-4{\cdot}8)$ (c) $5{\cdot}1-(-2{\cdot}3)$
(b) $-9{\cdot}5-(8{\cdot}1)$ (d) $-m-(-k)$

69. Find the values of the expressions $b+c$ and $b-c$, when :

(a) $b=2{\cdot}1$, $c=4{\cdot}5$ (c) $b=-5{\cdot}4$, $c=-3{\cdot}6$
(b) $b=-7{\cdot}6$, $c=6{\cdot}7$ (d) $b=2{\cdot}8$, $c=7{\cdot}4$

70. Find the value of the expression,

(i) $m-(n-c)$ when :

(a) $m=-1{\cdot}8$, $n=-5{\cdot}4$, $c=-4{\cdot}6$
(b) $m=0$, $n=-6{\cdot}7$, $c=-5$

(ii) $a+(b-a-c)$, when :
$a=b=-7$, $c=3{\cdot}25$

71. Find the value of the expression $a+(b-c)$, when :

(a) $a=-5{\cdot}25$, $b=3{\cdot}25$, $c=-8$
(b) $a=72$, $b=-4{\cdot}5$, $c=-6{\cdot}2$

72. Represent the fraction $-6{\cdot}87$ as a sum of two numbers, one of which is 36.

73. Represent the fraction $2{\cdot}8$ as a difference of two numbers one of which is equal to the number $7{\cdot}7$.

6. Solution of Equations and Problems

Properties of equalities, which were established for integers, are true for decimals also. Therefore, it is possible to use them, if necessary, for the solution of equations.

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74. Solve the equations :
(a) $x+2{\cdot}8=-3{\cdot}72$ (d) $m-6{\cdot}8=8{\cdot}7$
(b) $-4{\cdot}1+a=-20{\cdot}3$ (e) $-m+10=3{\cdot}76$
(c) $a+a+a=-8{\cdot}8+a$ (f) $x+x+x=-27{\cdot}9$

75. I have thought of a number. If to this number I add the same number and −2·5, then the result will be greater than the number by 2·2. Find the number.

76. I have thought of a number. If from the decimal 7·9 I subtract this number twice, the result will be less than the number by 2·6. Find the number.

77. Measure the segments AB and BC in fig. 3.12 and compare the results in the two cases.

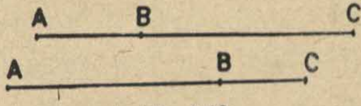

Fig. 3.12

78. The velocity of a steamer in still water is 21·6 km/h. The velocity of the stream is 4·9 km/h. Find the velocity of the steamer up and down the stream.

79. A motor-boat covers up-stream 16·5 km/h. Find the velocity of the motor-boat down-stream and its velocity in still water, if the velocity of the stream is 4·7 km/h.

80. The velocity of a boat in still water is 6·5 km/h. The velocity of the stream is 2·5 km/h. Find the velocity of the boat up and down the stream.

81. A customer had Rs. 21·3. At an emporium he spent Rs. 17·26, and at a food-store, Rs. 2·96. What amount of money is left with him ? Solve it in two ways.

82. Work out the equations :
(a) $y-4{\cdot}5=2{\cdot}56$ (b) $30-x=-79{\cdot}2$
(c) $7+7=3z+7{\cdot}1$

83. They brought some amount of wheat and rice to a godown. Rice was 4·7 tonnes less than wheat. What was the aggregate amount of rice and wheat brought to the godown, if they brought 27·4 tonnes of rice ?

84. Some tourists left the camp in the southern direction and covered 9 km, then turned to north and covered 22·3 km more, and at last turned to south and covered 14·4 km. How far from the camp did they halt ?

7. Approximate Numbers

A chairman was asked : 'How many people attended the meeting ?' He knew the exact number (316) but answered, 'above three hundred'. He had changed the number 316 to the 'more round' and, therefore, more convenient form for understanding. Let us consider another example. While preparing for a camp we took a rope of 21·65 m in length. But if we were asked about the length of the rope we would give some round number, for example, 21 or 22, instead of the number 21·65. Why ? Because these numbers are easily understood. Moreover, a few centimetres are not very important in this case.

These new numbers are called *approximate values* of the numbers we have spoken about.

So, 300 is an approximate value of the number 316.

21 is an approximate value of the number 21·65.

22 is also an approximate value of the number 21·65.

We could replace the number 21·65 by the number 20. In that case we'd get a new approximate value of the number 21·65. Replacing the number 21·65 by 20 as its approximate value we make an error equal to 1·65.

$$21{\cdot}65-20=1{\cdot}65$$

Thus, the difference between a number and its approximate value or between the approximate value of the number and the original number (let us always consider the positive difference) is called the error of the approximate value. Thus :

$$21{\cdot}65=20 \text{ (error } 1{\cdot}65)$$

It is read 'approximately equal to'

Find the errors in the approximate values 21 and 22 of the number 21·65.

85. A stone weighs 4·75 kg. If the number 4·75 is replaced by its approximate value 5, find the error. Write the approximate equality.

86. It was said that 3 thousand trees had been planted. But actually 3089 trees were planted. Write down the approximate equality.

87. A rectangle, having sides 30 cm and 25 cm, was shown in the classroom and a student was asked to find out its area. He said : '800 sq cm'. Find the error made by the student. Write down the approximate equality.

88. A month has 30 days. Write down the approximate equalities for September, January and February. What is the error in a year if the number of days is taken to be 30 for all the months ?

89. What natural number is to be taken as the best approximate value for the number 61·375 ? (best approximate value is that which has the least error.)

90. Choose among natural numbers the best approximations of the following decimals : 8·76; 54·246; 98·5.

8. Rounding of Numbers

The decimal 53·275 is between two successive natural numbers 53 and 54, *i.e.*,

$$53 < 53{\cdot}275 < 54$$

Take the number which has the smallest error and write down an approximate equality :

$$53{\cdot}275 > 53$$

Suppose we are given the decimal 58·672. The best approximation of it among the natural numbers is the number 59.

In previous examples we have replaced the decimals 53·275 and 53·672 by the natural numbers. We say that we have rounded the decimals.

In the first example we have taken a number which was less than the given number. In the second example we have taken the number which was greater than the given number.

What was the reason for that ?

Let us round the decimals : 23·13 ; 6·917 ; 862·49 ; 3·501 ; 6·5. Write the results in the table :

$23 < 23·13 < 24$	$23·13 > 23$
$6 < 6·917 < 7$	$6·917 < 7$
$862 < 862·49 < 863$	$862·49 > 862$
$3 < 3·501 < 4$	$3·501 < 4$
$6 < 6·5 < 7$	$6·5 < 7$

We saw that while rounding decimals to the whole numbers, we sometimes take the number which is greater than the given number and sometimes we take the number, which is smaller than the given number.

For that there is a very simple rule :

(i) If the first of the decimal digits is less than 5, we do not change the last digit of the integral part, just as the cases of 23·13 and 862·49 in which the approximate values after rounding were 23 and 862.

(ii) If the first of the decimal digits is either equal to 5 or greater than 5 we shall increase the last digit of the intergal part, just as in the cases 6·5, 6·917 and 3·501 in which the approximate values after rounding the decimals were 7, 7 and 4 respectively.

91. Round the following decimals to whole numbers and find the error : 8·76 ; 72·245 ; 4·5

92. Round the decimals to the tenths place :

(a) 67·309
(b) 4·901
(c) 305·02
(d) 0·861
(e) 85·5
(f) 902·02
(g) 0·38
(h) 99·811
(i) 859·5

93. Round the following numbers to whole numbers :
7·455 ; 45·67 ; 76·5 ; 102·25 ; 0·765 ; 0·439

94. What number of the row : ....4, 3, 2, 1, 0·1, 0·2 0·3, 0·4, ...is the best approximation of the decimals :
(a) 0·789 (b) 1·53 (c) 2·45 (d) 52·67

The decimal 8·425 is between two decimals
8·4 and 8·5 of the row : ......4, 3, 2, 1, 0·1, 0·2, 0·3, 0·4......
8·4 < 8·425 < 8·5

So, we have found among the numbers of this row the best approximations of the decimal 8·425.

The number 8·4 has less error, ∴ 8·425=8·4

The better approximations for the decimal 9·684 are the number 9·6 and 9·7. The smallest error gives the number 9·7.

∴ 9·684=9·7

Example 1. Round the decimal 3·8713 to the nearest tenth place. 3·8713=3·9

Example 2. Round the decimal 0·0826 to the nearest hundredth place. 0·826=0·08

Example 3. Round the number 3346·8 to the thousand.
3346·3=3000

95. Round to the hundredth the numbers :
0·07168 ; 0·0419 ; 10·081 ; 4·455

96. Round to the thousandth the decimals :
2·3143 ; 0·0419 ; 30·0045 ; 9·9995

97. Round to the nearest ten the numbers :
87·613 ; 214·3 ; 5264·85 ; 2959
Find the errors.

98. Round to the thousand the numbers :
8468 ; 29847·8 ; 305501 ; 279068

99. Round to the nearest tenth the decimals :
76·542 ; 86·55 ; 7·6845 ; 0·00009 ; 0·974

100. Round to the nearest thousandth the decimals :
7·666604 ; 0·22222 ; 6·5555

101. Round to the hundred the numbers :
7658 ; 674·8 ; 988·1 ; 67589 Calculate the errors.

102. Round to the million the numbers :
7629657·4 ; 870067006 ; 308986752456 ; 76201298706567

8. Multiplication

Multiplication of Decimal Fractions

To multiply a given number a by 3 means to find the sum of three numbers each of which is equal to the given number, i.e.,

[illegible]

[illegible]

[illegible]

Let us consider a few problems.

Problem 1. [illegible]

Solution: [illegible]

Problem 2. [illegible]

Solution: [illegible]

Problem 3. [illegible]

Solution: [illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Problem 4. [illegible]

Solution: [illegible]

1. [illegible]
2. [illegible]

107. One cubic centimetre of iron weighs 7·8g. Find the weight of a piece of iron if its volume is :
(a) 4·9 cubic centimetres
(b) 0·5 cubic centimetres

108. The length of a rectangle is 2·25 m. Find its area if the width is 0·96 m.

109. Multiply :
(i) 443 by 0·876 (ii) 7·06 by 8·06
(iii) 0·67 by 0·8 (vi) 0·3 by 0·24
(v) 80·8 by 4·05 (iv) 16·625 by 0·64

110. Express the following in square decimetres :
(a) 3·86 sq m (b) 0·6 sq m

111. One litre of oil weighs 0·92 kg. Find the weight of :
(a) 8·5 litres (b) 0·6 litres

112. Make operations
(i) $0{\cdot}18\times0{\cdot}25$ (ii) $0{\cdot}208\times0{\cdot}805$
(iii) $30{\cdot}01\times9{\cdot}03$ (iv) $64{\cdot}3\times100$
(v) $0{\cdot}00081\times1000$ (vi) $0{\cdot}006\times10$

113. Multiply and compare the product with the first factor :
(a) 84·6 and 9·5 (b) 84·6 and 1·05
(c) 84·6 and 1 (d) 84·6 and 0·05

Rules of multiplication of integers are also true for decimals. To multiply two decimals : (1) replace negative number by the positive, (2) multiply these numbers, (3) before the product put the sign '–' when factors have different signs and the sign '+' if the factor have the same sign. If one of the factors is zero then the produc is zero.

Example 1. Multiply $-2{\cdot}3$ by $0{\cdot}5=--(2{\cdot}3\times0{\cdot}5)=-1{\cdot}15$

Example 2. Multiply $-0{\cdot}7$ by $(-0{\cdot}6)=0{\cdot}7\times0{\cdot}6=0{\cdot}42$

The commutative, associative and distributive laws of multiplication are valid for decimals.

114. Multiply :
(a) $^{-}0{\cdot}8\times0{\cdot}6$ (b) $(^{-}0{\cdot}5)\times(^{-}0{\cdot}6)$
(c) $2{\cdot}1\times(^{-}0{\cdot}7)$ (b) $^{-}10\times(^{-}2{\cdot}65)$
(e) $^{-}0{\cdot}1\times(^{-}34{\cdot}3)$ (f) $445\times(^{-}4{\cdot}2)$

115. Find $0{\cdot}75$ times of the numbers $6{\cdot}34$, $^{-}86{\cdot}6$ and $^{-}100$.

116. Compute :

(a) $0{\cdot}5 \times (8{\cdot}616 - 9{\cdot}39)$

(b) $2{\cdot}4 \times (-7{\cdot}65 + 4{\cdot}35)$

(c) $(7 - 4{\cdot}919) \times 0{\cdot}7$

(d) $(8 - 10{\cdot}6) \times 4{\cdot}5$

117. Simplify :

(i) $^{-}2{\cdot}25 \times 0{\cdot}4\ ab$

(ii) $^{-}3{\cdot}7\ m \times 8\ n$

(iii) $5{\cdot}9\ ab \times (^{-}0{\cdot}5)$

(iv) $7{\cdot}4\ a - 26{\cdot}7\ a$

(v) $^{-}5{\cdot}6\ m - 3{\cdot}4\ m$

(vi) $6{\cdot}9\ n - n$

118. Find $6{\cdot}2$ times of each of the following numbers :

$^{-}7{\cdot}75$, $0{\cdot}9$ and $^{-}1000$

119. Find the value of the expression $a \times (m - n)$, when :

(a) $a = {}^{-}4{\cdot}2$, $m = {}^{-}5{\cdot}8$, $n = 0{\cdot}7542$

(b) $a = 3{\cdot}76$, $m = {}^{-}2{\cdot}56$, $n = {}^{-}200$

120. Find the value of the expression $(c - d) \times n$, when :

(a) $c = {}^{-}64{\cdot}2$, $d = {}^{-}34{\cdot}2$, $n = 11{\cdot}1$

(b) $c = {}^{-}9{\cdot}02$, $d = 0{\cdot}98$, $n = {}^{-}0{\cdot}4567$

121. Find the value of the expression $ab - xy$, when :

$a = {}^{-}8$, $b = 2{\cdot}2$, $x = {}^{-}5$, $y = 3{\cdot}25$

122. Solve the equations (by guessing or otherwise) :

(i) $3{\cdot}5a = 70$

(ii) $^{-}2{\cdot}8x = 0{\cdot}28$

(iii) $x \times x = 1$

(iv) $z \times z \times z = 0$

(v) $-5{\cdot}4p - 2{\cdot}6p = 8$

(vi) $25{\cdot}4k - 4{\cdot}4k = 210$

Division

Division of decimals has the same meaning as division of integers. To divide one number by the other means to find a third number, which when multiplied by the second number gives the first number.

Example 1. $0{\cdot}12 \div 0{\cdot}4 = 0{\cdot}3$, because $0{\cdot}3 \times 0{\cdot}4 = 0{\cdot}12$

Example 2. $^{-}2{\cdot}5 \div 0{\cdot}5 = {}^{-}5$, because $^{-}5 \times 0{\cdot}5 = {}^{-}2{\cdot}5$

Example 3. $^{-}8{\cdot}4 \div (^{-}4) = 2{\cdot}1$, because $2{\cdot}1 \times (^{-}4) = {}^{-}8{\cdot}4$

From these examples you can see that it is easy to estimate the sign of the quotient. Sometimes it is difficult to find the digits in the quotient. Let us examine the case when the divisor is a natural number.

Problem : 9·4m of woollen cloth is needed for 4 suits of the same size. How much cloth is needed for one suit ?

We see that the cloth needed for each suit is more than 2 m, as $4\times2=8$. Convert the remaining 1·4m into dm. We get 14 dm. So, besides 2 m for each suit we use 3 dm and 2 dm is left. Converting the remaining 2 dm into cm, we get 20 cm and 20 cm$\div$4 $=$5cm.

Finally for each suit the cloth used is 2m 3 dm 5 cm, that is, 2·35m. If we multiply the number 2·35 by 4, we get 9·4. So we have found the quotient of the number 9·4 when divided by 4, *i.e.*, $9{\cdot}4\div4=2{\cdot}35$.

Repeat once more the process of division. First of all the number 9 (the integral part of the decimal) is to be divided by 4 which gives quotient 2 and remainder 1. The remainder is converted into 10 tenths and the 4 tenths are added to it $(10+4=14)$. The sum 14 is divided by 4. We get 3 tenths as the quotient and the remainder 2 tenths. Then we convert 2 tenths into 20 hundredths and divide it by 4. The quotient is 5 hundredths.

The process of division can be written in the form given below :

$$\begin{array}{rl}
4\,\overline{)\,9{\cdot}4} & (\,2{\cdot}35 \\
-8 & \\
\hline
14 & \\
-12 & \\
\hline
20 & \\
-20 & \\
\hline
0 &
\end{array}$$

So, the division of the decimal by a natural number is carried out as the division of natural numbers. Decimal point is to be placed after the division of the integral part of the decimal.

Suppose, we are asked to divide 3·328 by 1·3. Multiply the divisor by 10 (to get a natural number). For not changing the quotient we have to multiply the dividend also by 10.

```
13 ) 33.28  ( 2·56
    −26
    ----
      72
     −65
     ----
       78
      −78
      ----
        0
```

123. Two kg of cheese costs Rs. 5·2. How much does 1 kg cost ?

124. A rope 25·76 m long has been divided into 8 equal parts. Find the length of each part.

125. 7 cu cm of silver weighs 73·5 g. Find the weight of 1 cu cm of silver.

126. Find the width of a rectangle, if its length is 24 cm and area is 86·4 sq cm.

127. Divide and verify the results :
(a) 1 by 8 (b) 0·3741 by 43 (c) 2·6 by 208

128. Divide and draw the conclusions :
(a) 32·4÷10 (b) 85·6÷100
(c) 7·26÷10 (d) 245·96÷1000
(e) 0·03÷100 (f) 5÷1000

129. Find the values of the following expressions :
(a) 54·0204÷4·2 (d) 0·45716÷0·22
(b) -11·648÷5·6 (e) 2·8÷0·14
(c) 3·507÷3·5 (f) 3÷0·75

130. Solve the equations :
(a) 0·14k=−0·287 (c) 7·254÷d=1·8
(b) −30·6p=185·436 (d) m÷2·05=−40·6

131. What is the speed of a man if he covers 11·5 km in 2·5 hours ?

153. Two pumps, having been switched on simultaneously, fill a pool in 85 hours. The capacity of the first pump is $16 m^3/h$ more than that of the second one. Find the capacity of the first pump, if in all $2890 m^3$ of water was pumped into the pool.

154. A scooter is following a lorry which is 27 km ahead of it. The velocity of the scooter is 1·375 times that of the lorry. Find the velocity of the scooter, if it overtakes the lorry in 1·8 hours.

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CHAPTER IV

FORMULAS

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1. Formula and Rule

USE OF FORMULAS IN CALCULATION

Using letters is very convenient for expressing the rules in a short form. Let us examine the following example :

For converting ares into sq m, we use the following rule : The number, expressing area in ares, is multiplied by 100 and the product so obtained expresses the same area in sq m.

But this rule may be expressed in a shorter *form* If a is the area expressed in ares, and b is the same area in sq m, then $b=100a$.

Suppose, we want to express the area of a plot of 5·2a in sq m. Since $a=5{\cdot}2$, we find b by the use of the formula :

$$b=100\times 5{\cdot}2=520$$

The area of this plot is 520 sq m.

An expression composed of numbers, letters, signs of operation and signs of equality or inequality is called a formula.

A formula is a substitute for a verbal rule. With the help of a formula we express the rule more shortly.

A formula, expressing the relation between certain quantities should be followed by an indication of the letters standing for the corresponding quantities and their units of measurement.

A square is given in fig 4·1 ; a is the length of its side (in centimetres) and p is its perimeter (in centimetres) then : $p=a+a+a+a$, *i.e.*, $p=4a$.

Read the rule of finding the perimeter of a square ? (refer to the deduced formula.)

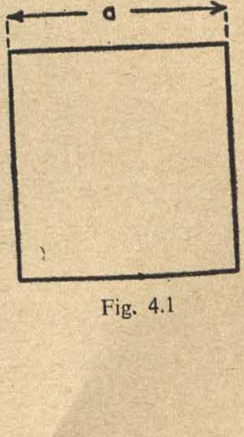

Fig. 4.1

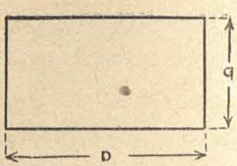

Fig. 4.2

1. In fig. 4·2, a rectangle is given. a is its length, b is its width and p is its perimeter (all are measured in centimetres).

   Work out the formula for finding the perimeter of the rectangle, and state the corresponding rule in words.

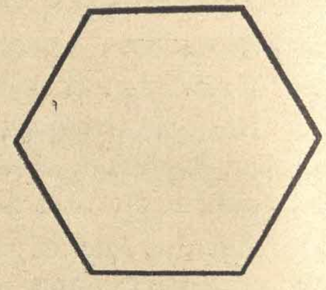

Fig. 4.3

2. To find the perimeter of a polygon with equal sides (fig. 4·3), it is necessary to multiply the length of one side by the number of the sides.

   Express this rule by a formula, introducing all the necessary notations.

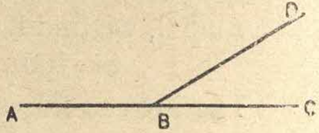

Fig. 4.4

3. Suppose, $\angle \mathbf{DBC} = a^\circ$ (fig. 4·4). What is the measure of $\angle \mathbf{ABD}$ in degrees ? Work out a formula for finding the measure of $\angle \mathbf{ABD}$. State the rule for finding the measure of one of the adjacent angles if the measure of the other angle is given.

4. Given the following sequence of numbers :

   $^{-}16, ^{-}13, ^{-}10, ^{-}7, ^{-}4,$ ……………………………………………………

   Express the first five numbers of this sequence by points on a number-line. On this line mark the point corresponding to the sixth term of the sequence. (Suppose that the regularity discovered by you is true for all the terms of the sequence.)

   Denoting one of the terms of the sequence by a, and the following term by b, express b in terms of a.

5. A sequence is composed in the following way : If a is a term of this sequence, and b the following term, then $b = {}^{-}2a$. State the rule according to which each term can be got from its previous term. The first term of the

sequence is equal to 2 ; find the next five terms. Represent the first six terms of this sequence by points on a number-line. Mark several points on the line corresponding to the terms of the sequence.

6. The weight of a cow can be found approximately by the formula : $P=\frac{lk}{50}$ in which p is the weight of the cow in kg, l is the length of the back, from the withers to the tail in cm and k the circumference near the bottom-chuck in cm.

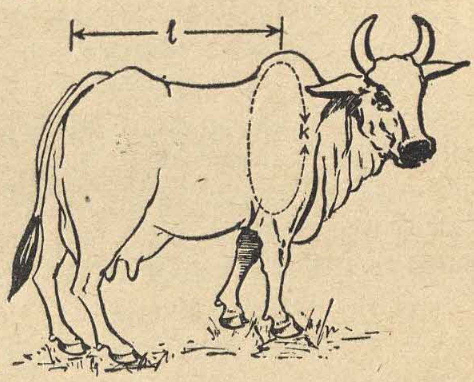

Fig. 4.5

Find the weight of the cow, if :

$l=112$ cm k=152 cm

$l=104$ cm k=136 cm

7. Suppose, S is the area of a rectangle in sq m, a is its length in m and b its width in m. Write the formula for finding the area S of the rectangle. Find S if :

(a) a=12m b=17 m

(b) a=3m 20 cm b=2m 50 cm

(c) a=160 cm b=210 cm

What are the units of measurement of S ?

8. Two rectangles have equal bases (1·8 cm each). The height of the first rectangle is 3 cm more than that of the second one. Find the height of each of the rectangles, if it is given that the sum of their areas is equal to 12·6 sq cm.

9. The temperature of air being the same, F is the number of degrees on the Fahrenheit-scale while C is the number

of degrees on Centigrade-scale. If F is given, C can be found by the formula :

$$C=\frac{5(F-32)}{9}$$

Find C (correct to one degree), if $F={}^{-}12$, 0, 17, 32, 50 and 76. Write down the results in the form of a table.

Note that the positive readings on the Fahrenheit-scale can correspond to both positive and negative readings on the Centigrade-scale. For example, 50°F will be positive on the Centigrade-scale but 14°F will be negative on it. What temperature on the Fahrenheit-scale corresponds to 0°C ? What temperature in Centigrades corresponds to 0°F (refer to the table drawn up previously) ?

10. Given a square, a triangle and a circle, we can arrange them in different ways (fig. 4·6).

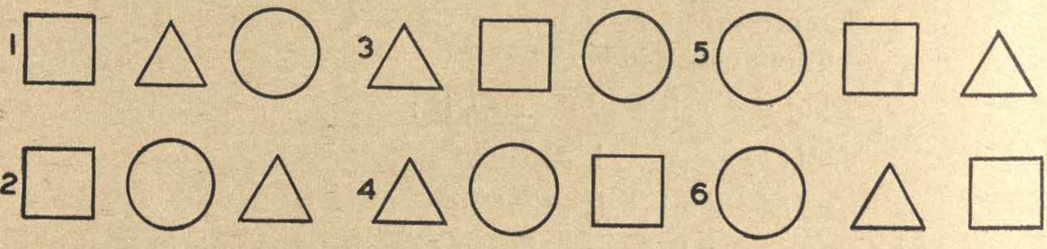

Fig. 4·6

Each arrangement of these objects is known as a *permutation.* We see the three objects being arranged in 6 permutations.

11. Suppose today's time table for class V contains five periods : Hindi, Mathematics, Biology, History, Physical-exercises. How many different time tables are possible to work out for this day ?

Formulate a rule of multiplying a number by the sum of two numbers.

This rule can be written as the equality :

$c(a+b)=ca+cb$

Now let us *interchange* the right and the left parts of this equality :

$ca+cb=c(a+b)$

In this case it is said that we have read the equality $c(a+b)=ca+cb$ 'from right to left'. You know that it is a distributive property. Using the distributive property, we formulate the rule to factor expressions, terms of which have the common factor.

12. Calculate :

(a) $-8{\cdot}471 \times 0{\cdot}3 \times 1{\cdot}7 - 1{\cdot}7 \times 0{\cdot}129 \times 0{\cdot}3$

(b) $-6{\cdot}91 \times 0{\cdot}017 - 0{\cdot}017 \times 5{\cdot}09 + 0{\cdot}017 \times 4$

13. Check whether the following equalities are true :

(a) $(100 \times 4) \div 2 = (100 \div 2) \times 4$

(b) $(-18 \times 0{\cdot}17) \div 6 = (-18 \div 6) \times 0{\cdot}17$

(c) $\frac{15 \times 0{\cdot}037}{0{\cdot}5} = \frac{15}{0{\cdot}5} \times 0{\cdot}037$

Prove the above equalities with the help of the following rule. To divide the product of two factors by a number (not equal to zero) it is sufficient to divide one of the factors by this number and then multiply the quotient thus obtained by the second factor.

This rule can be written down in the form of an equality :

$$\mathbf{\frac{ab}{c} = \frac{a}{c} \times b (c \neq 0)}$$

Now let us prove that this equality is true : Suppose $\frac{a}{c} = r$, then $a = cr$ and so $ab = (cr)\,b$ (by the associative law of multiplication).

We take the case, when a quotient is a whole number or a finite decimal. If as a result of dividing a by c, we get infinite decimal, we can take this equality as an approximate equality.

Later on we prove :

$\frac{ab}{c} = \frac{a}{c} \times b$ for any value of a, b and c $(c \neq 0)$

$ab = c(rb)$. Thus by the definition of division.

$\frac{ab}{c} = rb$; but $r = \frac{a}{c}$, *i.e.*, $\frac{ab}{c} = \frac{a}{c} \times b$

Read the last equality from right to left, and state the rule of multiplying a quotient by any number.

14. One mile is approximately equal to 1·6 km. Work out the formula for converting miles into km.

15. To send a telegram, we pay 10 p for each word. Denoting a as the cost of telegram (in p), and n as the number of words, work out the formula for determining the cost of a telegram. Find a, if $n=8$.

16. Each term of a sequence starting from the second is got by multiplying the previous term by 3. Suppose, a is a term of the sequence and b is a term following a, express b in terms of a. The 12th term of this sequence is equal to $^{-}15$. Find the 11th, 13th and 14th terms.

17. According to physicians the youngsters below 18 must normally sleep for b/h day where b is given by the formula $b=8+\frac{18-t}{2}$ (where t is the age in years). Find how many hours of sleep should a child of 12 months old, 2 year old and, 3 year old have ? Represent the result in the form of a table.

18. The speed of sound propagation in the air at a temperature of t degrees is given by the formula $V=331+0{\cdot}6\,t$ where V is the speed of propagation (in metres per second), t is the temperature of the air (in centigrades). Find V, if $t=0$, 18 and 32.

19. Metal rods expand on heating. If at 0° the length of a rod is l cm; and at t° is c cm, then $c=l\,(1+at)$ Coefficient of expansion 'a' depends on the rod. Find c, when :

(a) $l=500$, $a=0{\cdot}000018$, $t=100°$

(b) $l=80$, $a=0{\cdot}000012$, $t=150°$

20. Given four objects in fig. 4.7, find the number of all

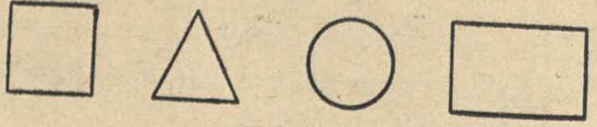

Fig. 4.7

possible permutations. Try to make permutations as made in example 12.

21. The length of a rectangle is thrice its width. Find its sides, if the perimeter of the rectangle is 2 cm more than the perimeter of a square with each of its sides equal to 4 cm.

22. Find the value of each of the following expressions :
    (a) $(-0{\cdot}017-0{\cdot}083-0{\cdot}0) \times (-1{\cdot}34-2{\cdot}12)$
    (b) $(2{\cdot}713-1{\cdot}008-0{\cdot}114) \div 1{\cdot}5$ (nearest to ·1)
    (c) $(17{\cdot}23\times 6{\cdot}07-17{\cdot}23\times 7{\cdot}93+17{\cdot}304$
    (d) $(2{\cdot}45\times 0{\cdot}14-0{\cdot}14\times 5{\cdot}15) \div 0{\cdot}075$
    (e) $3{\cdot}6\times(-0{\cdot}17-0{\cdot}4)-2{\cdot}4\times(2{\cdot}1-3{\cdot}4)$
    (f) $(-6{\cdot}12-4{\cdot}15\times 1{\cdot}06)\div(-1{\cdot}1)$
    (g) $(136{\cdot}1+207{\cdot}4)\div 5{\cdot}6$ (nearest to ·01)

2. Working out Formulas (Averages)

So far we made calculations using given formulas. Now let us try to work out some simple formulas.

Example : A boy kept on counting the number of steps he was walking on his way to the school for 5 days. The results were : 1260, 1262, 1259, 1263 and 1257 steps respectively.

Now let us determine the *average* number of steps the boy walked on his way to the school daily. Adding the numbers of addends and dividing the sum by 5, we will get :

$$\frac{1260+1262+1259+1263+1257}{5}=1260{\cdot}2=1260 \text{ (approximately)}$$

Thus, the boy on an average walked 1260 steps daily on his way to the school. It is said that we have found the arithmetic mean of the numbers 1260, 1262, 1259, 1253, and 1257. *The average or arithmetic mean of a number of quantities is the quotient obtained by dividing the sum of these quantities by their number.*

23. Find the arithmetic mean of:
    (a) 1·6, 1·1, 0·3
    (b) 12·06, 15·14, 3·96, 14·11
    (c) 12·3, 12·4, 11·9, 12·1, 82·6

24. Suppose a and b are two numbers. Work out an expression for the arithmetic mean of these numbers, and find its value, when $a=10$; $b=18$. Represent these numbers and their average by points on a number-line. Is there any peculiarity in their arrangement ?

25. Find the arithmetic mean of the numbers $^{-}12$ and $^{-}1$. Represent these numbers and their arithmetic mean by points on a number-line. Is there the same regularity in their arrangement on the number-line as in question 24 ? Is it the same with the numbers $^{-}10$ and 2 ?

26. Suppose a, b, c, and d are four numbers and m is their arithmetic mean. Work out the formula for finding m.

27. Suppose we have n numbers : $a, b, c, \ldots\ldots$ Find a formula for finding y, where y is the arithmetic mean of these numbers.

28. The sides of a rectangle are a cm and b cm and its perimeter is p cm. Three pupils worked out the following three different formulas for finding the perimeter :
(a) $p=a+b+a+b$ (b) $p=2(a+b)$ (c) $p=2b+2a$
Which of these formulas is correct ?

29. The perimeter of a rectangle is equal to 12·4 cm and its length is 3 cm more than its width. Find the sides of the rectangle.

30. Suppose a is the length of a rectangle (in metres) and b is its width (in metres) and S is its area (in ares). Express S in terms of a and b.

31. Introduce the necessary notation and work out the formula for calculating the shaded areas of the fig. 4.8.

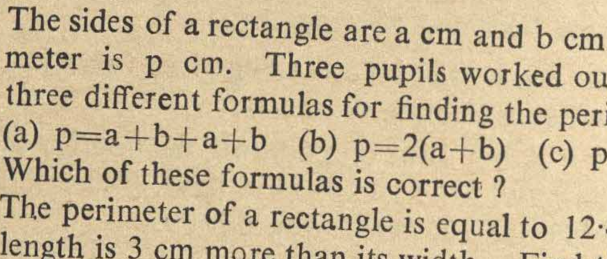

Fig.

Measure (nearest to 1 cm) and calculate the area by the formula (nearest to 1 cm^2).

32. There are a kg of biscuits and b kg of sweetmeats. Explain the meaning of the expression : $a+b$ and $a-b$.

Find m and n, if $m=a+b$; $n=a-b$, in the following cases :

1. $a=37{\cdot}5$; $b=12$
2. $a=205$; $b=120$

33. There are 'a' poles along the road. What is the total number of intervals between the neighbouring poles ?

34. A tourist leaves town A, to reach at the tourist camp D. The distance between A and D is 250 km. The tourist covers a part of this distance on foot, moving at a speed of 5 km/h, and the remaining part of the distance in a car moving at a speed of 60 km/h Had he covered the entire distance by car, he could have reached the camp D 2 hours earlier. For how many hours did the tourist walk ? In how many hours did the tourist cover the distance between the town and the camp ?

Example : From two towns (fig. 4.9), which are S km apart, 2 trains started simultaneously. They are approaching each other, the first starts with a speed of V km/h and second with a speed of W km/h. They met after t hours. Express S in terms of V, W and t. Examine the solution of the problem given below and answer the question.

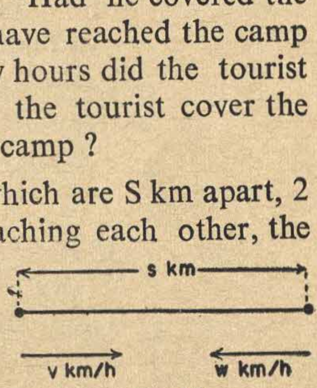

Fig. 4.9

Solution :

Vt (km)=the distance, covered by the first train before it meets the second.

Wt (km)=the distance, covered by the second train before it meets the first.

The sum of these distances is equal to S km, *i e.*, $S=(Vt+Wt)$.

Taking out the common factor t, we get : $S=(V+W)\times t$.

What is the meaning of the expression $(V+W)$?

What reasoning can directly lead to the formula
$S=(V+W)\times t$?

35. A cyclist starting from the village A is covering the distance between the village A and the town B with a speed of a km/h and another from B with a speed of b km/h. Five hours after they started, the distance between them was d km. Express d in terms of s, a and b. Find the value of d, when $a=12$ km, $b=10$km and $s=180$ km.

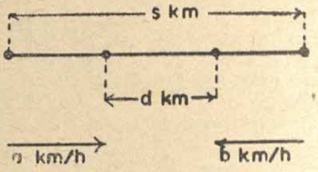

Fig. 4.12.

36. A cyclist leaves the town A, with a velocity of w km/h. (fig. 4.11). At the same time a motor-cyclist leaves the village B, in the same direction which is s km distant from the town A, with a velocity of v km/h. The motor-cyclist overtakes the cyclist in t hours. Express s in terms of v, w and t. Examine the solution given below and answer this question :

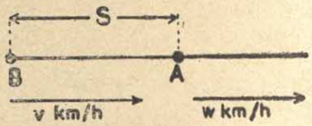

Fig. 4.11

Solution :

Vt (km) is the distance covered by the motor-cyclist,
wt (km) is the distance covered by the cyclist.
The motor-cyclist covers a distance of s km more than the cyclist.

$\therefore\ s=vt-wt$

or $s=(v-w)\,t$

What is the meaning of the expression $(v-w)$?
What would have happened in case $v=w$?

37. Ram was going to the school 1·5 km away from his house and was moving with a velocity of 60m/minute. At the same time Suresh also left his house situated 2 km away from the school and was moving with a velocity of 80m/minute in the same direction. How much time will Suresh take to overtake Ram ? Will Suresh be able to overtake Ram or not before reaching the school ?

38. Find the arithmetic mean of the numbers : $^{-}13$, $^{-}1$, 4 and 7. Represent these numbers and their arithmetic mean on a number-line.

39. Find the arithmetic mean of the following numbers :

(a) 0·367, 1·844, 5·367

(b) 1316, 1318, 1307, 1309, 1305, 1310, 1311

(c) 12·65, 13·67, 17·08, 12·15

40. The area of a rectangle is S square decimeters, its length is a m and width b m. Express S in terms of a and b.

41. One side of a triangle is a cm, the other two sides are b cm each and its perimeter is p cm. Deduce the formula for finding the perimeter.

42. The perimeter of a triangle is 46 cm. The first side is 4 cm longer than the second, and 2 cm shorter than the third. Find the sides of the triangle.

43. Deduce the formulas for finding the velocities of a ship, down-stream and up-stream, if the velocity of the ship in still water is a km/h and that of the stream is b km/h. What would happen if $a=b$, $a<b$?

44. A motor-boat covers the distance of b km, down-stream in 7 hours, and up-stream in 9 hours. Find the velocity of the motor-boat in still water ; the velocity of the stream is 2 km/h.

45. Two trains depart simultaneously from two different stations 390 km apart. The first train is moving with a velocity 1·5 times that of the second. Find the velocities of the trains, if it is given that having been 4 hours in motion there still remained 30 km to be covered before they could meet.

46. At 12 noon a pupil left his home for a village and was moving with a speed of 4·5 km/h. Two hours later his father followed him at the rate of 6 km/h. At what time will the father overtake his son ?

3. Transformation of Formulas

Sometimes it is very useful to get new formulas from the given formulas. Solve the following questions to understand it.

47. Find I by the formula

$$I=\frac{P\times r\times t}{100} \text{ when } P=400,\ r=5,\ t=2$$

48. Transform this formula to get expressions for (i) P, (ii) r and (iii) t. Find the value of I in the above question by taking four different values for P, r and t respectively. Also find (i) P, (ii) r and (iii) t in each of these cases.

49. a and b are two numbers and c is their arithmetic mean. Express c in terms of a and b. From this formula find the expression for a in terms of c and b.

50. A body is moving with a constant velocity of v m per second. If s m is the distance covered by the body in t sec ; express s in terms of v and t. Find v in terms of s and t and then express t in terms of s and v.

51. $a\div b=c$. Express a in terms of b and c. What number cannot be a divisor ? How can division be checked ?

52. Suppose a, b, c and r are some natural numbers, where a is the dividend, b the divisor, c the quotient and r the remainder. Express a in terms of b, c and r. Transform this formula to get expression for c. Find the formula for the divisor in terms of the dividend, quotient and remainder.

53. From the formula $c=b(1+at)$, find an expression for t.

54. Add 5 to twice the number of your house. Multiply this sum by 50 and then add 1714 to the product. From this sum subtract the year of your birth. Taking the last two digits, you get a two-digit number which indicates your age in 1964, and the number after removing the last two digits is the number of your house. Check it.

Why does it happen like that ? We can answer this question, if we write down the solution of an expression and then simplify it.

Suppose a is the number of your house and let us assume that your year of birth is 1956.

If we carry out the processes mentioned above, the final number b that we get is :

$$(2a+5)\times 50+1714-1956$$
$$=(100a+250+1714)-1956$$
$$=(100a+1964)-1956=100a+(1964-1956)$$

Therefore : $b=100a+(1964-1956)$. Now $1664-1956$ was your age in 1964. The number of hundreds is the number of your house. Guess what should be the condition of the problem if you would like to get your age in 1965 and 1967 ?

Example : Two motor-boats are nearing each other. One of them is moving down-stream with a velocity of v km/h and the second up-stream with a velocity of w km/h. In one hour both of them cover s km. If the velocity of each motor-boat in still water is x km/h and the velocity of the stream is y km/h, find s, if $x=21$ and $y=1{\cdot}7$

Let us solve this problem in two ways :

First method :

$$v=x+y=21+1{\cdot}7=22{\cdot}7$$
$$w=x-y=21-1{\cdot}7=19{\cdot}3$$
$$s=v+w=22{\cdot}7+19{\cdot}3=42$$

Second method : $s=v+w$

Substitute v for $x+y$ and w for $x-y$ and simplify the expression.

$$s=v+w=(x+y)+(x-y)$$
$$=x+y+x-y=2x$$

Calculations in the second case are easier.

Note that in solving this problem by the second method we get the result independent of y, *i.e.*, to find the distance covered by the motor-boats in 1 hour, one need not know the velocity of the motor-boats in still water.

55. $a=m+n-c$; $b=m-n+c$

Find the sum of the numbers a and b, when $m=0{\cdot}016$; $n=3{\cdot}147$, and $c={}^{-}0{\cdot}184$. Find whether all the data are necessary.

56. In the formula $m=7x+3y$ substitute x for $y-1$.

57. Substitute the values given in the brackets against each question and simplify the expressions thus obtained.
    (a) $1+a-b$ $(a=-t\ ;\ b=1+2+t)$
    (b) $2A+B$ $(A=x-1\ ;\ B=3x+1)$
    (c) $5M-4N$ $(M=2y-3\ ;\ N=y+1)$

58. Draw up a table of the values of the expression : $A=5\times(t-1)+7$, when $t={}^{-}3{\cdot}4$, ${}^{-}1{\cdot}8$, $0{\cdot}2$. Mark the values of A thus obtained, by points on a number-line.

59. Solve the equation $ax-3{\cdot}6=-4{\cdot}5(a\neq 0)$

60. Find y, when $y=-3{\cdot}17\times(x-2{\cdot}16)$ and $x=8{\cdot}36+2{\cdot}16$

61. From the formula $y=\frac{k}{x}$ $(x,\ y\neq 0)$ derive the formula for k in terms of x and y ; and derive the formula for x in terms of k and y.

62. If $s=vt+wt$ find it, when $v=0$, $w=0$. Express t in terms of s, v and w.

63. First divide, then check it up by multiplying :
    (a) $5a\div a(a\neq 0)$ (c) $\frac{12ax}{x}(x\neq 0)$
    (b) $bm\div m(m\neq 0)$ (d) $\frac{3ab}{ab}(a\neq 0,\ b\neq 0)$

64. Simplify the following and in each case express the dividend in terms of the divisor, the quotient and the remainder.
    (a) $316\div 21$ (b) $15\div 26$ (c) $7236\div 36$

65. Think of a whole number, double it, add 30, multiply the sum by 3, subtract 90 from the product and divide the result by 6. The result that you get is the number you thought of. Write down the solution of the problem in the form of an algebraic expression. Simplify and explain the result.

66. $a=x+y\ ;\ b=x-y$. Find the sum of a and b, when $x=0{\cdot}1$; $y=-0{\cdot}34786$. Are the entire given data necessary ?

67. In the expression $5x-7y$, substitute $(t+2)$ for x and $(2t-1)$ for y and simplify.

4. Raising to Power

Formulas Containing Exponents

NOTION OF POWER: While solving some problems, sometimes we may get the product of several equal factors. Here are two problems to be examined.

68. Suppose the side of a square is a cm and its area is S cm^2. Express S in terms of a. Find S, when $a=1{\cdot}2$

69. The edge of a cube is a cm and its volume is V cm^3. Express V in terms of a. Find V when $a=4$.

The product of a number of equal factors is generally written in a shorter way as follows: $3\times3\times3\times3=3^4$ (three raised to the power of 4)
$8\times8\times8=8^3$ (eight raised to the power of 3),
$2^5=2\times2\times2\times2\times2$ (product of five factors, each equal to 2), $(^-3)^4=(^-3)\times(^-3)\times(^-3)\times(^-3)$ (product of 4 factors, each equal to $^-3$).
$3^6=3\times3\times3\times3\times3\times3$ (product of six factors, each equal to 3).

In the last example the number 3 is called the *base* and the number 6 is called the *exponent*. The exponent indicates the number of times the base is taken as a factor in the product. A number equal to 3^6 is called a *power* of 3. In this case it is the sixth power of three.

Examples: (a) $2^5=32$, 2 is the base and 5 is the exponent 2^5 or 32 is the fifth power of the number 2.

(b) the formulas for the area of square or the volume of a cube, deduced in problems 68 and 69, can be written as:
$S=a^2$ (a raised to the power two, or 'a raised to the second power')
$V=a^3$ ('a raised to the power three' or 'a raised to the third power').

From these examples, it is clear why the second power of a number is (sometimes) called as the square of the number and the third power as the cube of the number. The words 'squared' and 'cubed' originated with the use of these operations to obtain respectively the areas of squares and volumes of cubes. Instead of 'a raised to the second power' we say 'a squared' ; instead of 'a raised to the power three', we say 'a cubed' or 'cube of a'. The exponent is omitted for the first power :

$2^1=2, (-1{\cdot}3)^1=-1{\cdot}3$

70. Calculate :

    $61^3, (-3)^5, (1{\cdot}44)^4, 2^7, (1{\cdot}95)^3, (3{\cdot}2)^5$

71. Find the values of the expressions $5a$ and a^5, when $a=3{\cdot}2, -3$
72. Draw up a table of squares of the natural numbers from 1 to 20.
73. Draw up a table of cubes of the natural numbers from 1 to 10.
74. Calculate 10^n, when $n=1,2,3,4,5,6$. What changes does the number which is the power of 10 undergo with the increase of the exponent ?
75. Calculate $(0{\cdot}1)^n$, when $n=1,2,3,4,5,6$. What changes does the number which is the power of 0·1 undergo with the increase of the exponent ?
76. Compare the expressions :

    (a) 10^2 and 2^{10} (b) 2^4 and 4^2 (c) 3^4 and 4^3

77. a and b are natural numbers. Is it in general true that $a^b=b^a$? Give some values to a and b and see the result.
78. Calculate and draw the possible conclusions :

    (a) $1^4, 1^7, 1^{12}, 1^n$ (n a natural number)

    (b) $0^5, 0^9, 0^{11}, 0^n$ (n a natural number)

Power of a Negative Number

Suppose we are asked to calculate $(-3)^5$. Try to find at first sight the sign of the result.

Calculate :

$(-2)^2, (-5)^5, (-7)^2, (-1)^5$

We see that raising a negative number to a power with an even exponent gives us a positive number, while raising a negative number to a power with an odd exponent gives us a negative number.

79. Find which of the following numbers are positive, and which are negative :
$(-0{\cdot}2)^9, (3{\cdot}1)^5, (-7)^2, 3^7, (3{\cdot}16)^2, (-1)^4, (2{\cdot}07)^3(-1)^{11}$ and $(0)^8$
80. Which is greater ?
(a) $(-3{\cdot}1)^5$ or 0 (b) 0 or $(-2{\cdot}4)^4$
(c) 7^4 or $(-5)^7$ (d) 1^4 or $(-2)^4$
(e) 1^5 or $(-2)^5$
81. If $a^7=0$, can a be a negative number ?
82. If $a^6=0$, which of the relations are true ?
$a<0, a>0, a=0$
83. Can the inequality $x^2>x$ be true when :
(a) $x>0$, (b) $x<0$ and (c) $x=0$?
84 Is it always true that $x^2=(-x)^2$?
85. Mark points on a number-line corresponding to all the values of x in whole numbers satisfying the inequality $1<x^2<110$.

Order of Operations in the Expressions containing Power

We use the following order of operations : First simplify the quantities raised to a power ; then simplify the quantities involving multiplication and division in the order in which they are written and then simplify the quantities involving addition and subtraction, as they occur. If you have brackets, first do all the operations in brackets and then remove the brackets.

Example : Calculate the value of the expression $(-8)^3-2^2\times 3$.

(1) $(-8)^3=-512$ (3) $4\times 3=12$

(2) $2^2=4$ (4) $-512-12=-524$

In a shorter way it can be written as :

$(-8)^3-2^2\times 3=(-512)-4\times 3=-512-12=-524$

86. Calculate :

(a) $-8^3+(-2)^2\times 3$ (c) $(-2-3)^3$

(b) $(-2)^3+(-3)^3$ (d) $-(2+3)^3$

87. What will be the order of operation to calculate the value of each of the following expressions (suppose the values of the letters are given) :

(a) $(2x)^3-1$ (b) $2x^3-1$ (c) $(2x-1)^3$

(d) a^2-b^2 (e) $(a-b)^2$ (f) a^3-2b^2

88. Write down the numbers, given in the following sentences, in the form of the product of a number and the power of ten, according to the following pattern :

$5000=5\times 1000=5\times 10^3$

$12000000=12\times 1000000=12\times 10^6$

(a) the distance between the Earth and the Sun is 150000000 km

(b) the population of India is approximately 430000000

(c) out of the Earth's surface totalling 510000000km², land occupies 149000000km² and water occupies 361000000km²

(d) a computer does 7200000 operations per hour

(e) 1 cm³ of gas contains 27000000000000000000 molecules.

89. Calculate and read out :

36×10^5, $4{\cdot}5\times 10^8$, $14{\cdot}>10^9$

90. $y=2ab^2$. Calculate y, when $a=-1{\cdot}2$; $b=-1{\cdot}3$

91. For approximate calculation of the volume of a haystack

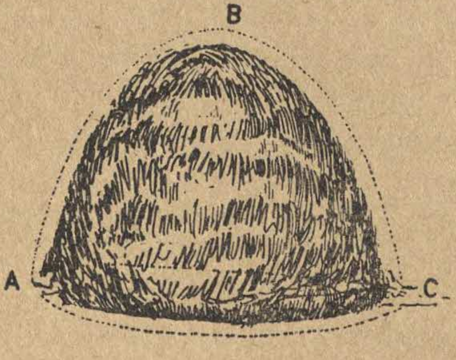

Fig. 4.12

(fig. 4.12), we use the formula :

$$V=\frac{nl^2}{12}$$

where V is the volume of the haystack (in cubic metres) l the length of the circumference of its base (in metres). n the length of ABC (in metres). Calculate the volume of haystack when : (a) $l=12$ m, $n=18$ m (b) $l=132$ dm, $n=154$ dm

92. A stone falling on the ground, covers the distance of s m in t seconds where s is given by $s=4{\cdot}9t^2$. Calculate s, if $t=3,8,5,20$

93. The length of a rectangular parallelepiped is a cm ; the width is equal to the length and its altitude is b cm and S (cm^2) is the area of the total surface of the parallelpiped. Express S in terms of a and b.

    Find S when $a=12$, $b=15$.

94. Find the value of the expression a^n, when :

    (a) $a=-0{\cdot}036$, $n=2$ (b) $a=84{\cdot}5$, $n=1$

    (c) $a=-1$, $n=8$ (d) $a=0$, $n=9$

95. The speed of light is 3×10^5 km/sec. Calculate and read the result.

96. Compare the expressions :

    (a) 3^3 and 3^5 (d) $(-0{\cdot}2)^3$ and $(-0{\cdot}2)^5$

    (b) $(-3)^3$ and $(-3)^5$ (e) $(-3)^3$ and $(-3)^2$

    (c) $0{\cdot}2^2$ and $0{\cdot}2^5$ (f) 3^5 and 5^3

97. (a) Which is greater, $2a$ or a^2, if $a>0$?

    (b) $b>0$, which of the inequalities is true :

    $b^5<0$, $b^5>0$ or $b^5=0$?

98. Find the number, square of which is 81. How many numbers can you find ?

99. $10<x^2<20$ and x is a whole number. Find x.

100. Mark on a number-line all the whole numbers satisfying the inequality $x^2<30$.

101. Find the value of the expression :

$$\frac{(m^2+n^2)+2mn}{5} \text{ when } m=-0{\cdot}6,\ n=-0{\cdot}4$$

102. $$y=\frac{(x^2+6x+9)-(x^2-6x+9)}{12}$$

Fill in the table :

x	-2	$-1{\cdot}2$	-1	0	$1{\cdot}2$
y					

103. $s=\dfrac{(a^2+2a)-(a^2-2a)}{a}$. What is the value inappropriate for a ? Fill in the table.

a	$-0{\cdot}5$	-3	1	100	1000
s					

104. The edge of a cube is equal to a cm and its total surface is S cm^2, Express S in terms of a. Find S when $a=1{\cdot}3$.

105. The base of a rectangular parallelepiped is a square with the side equal to a m, the altitude of the parallelpiped is h m and its volume is Vm^3. Express V in terms of a and h. Find h using the formula obtained.

6. Ratio and Proportion

106. Calculate how many times a rope of 20 m is that of a rope of 5 m in length ?

107. Calculate what part of one metre is 20 cm ?

We found the answers to the questions given above by division. In order to compare two given values, it is necessary to divide one of them by the other or, as it is said, we have to find the ratio of these values.

Definition. **The ratio of two similar values is the quotient obtained by dividing one of them by the other** (by similar values we mean the units of the same kind such as units of width, length and money etc).

108. Find the ratio of :

(a) 2 kg to 1 tonne (b) 1·5 h to 20 mts

(c) a sq cm to a sq m (d) 7 to 35
(e) number a to number b ($b \neq 0$)

What does each of these ratios illustrate ?

We can write the ratio of the number a to the number b as : a : b or $\frac{a}{b}$

109. Multiply simultaneously both terms of the ratio of numbers 20 and 50 by the same number (not equal to zero) and calculate the result.

Divide simultaneously both terms of this ratio by the same number (not equal to zero) and calculate the result.

Compare the results obtained in both the cases and draw the corresponding conclusion.

Conclusion : **A ratio will not change, if all its terms are multiplied or divided by the same number (not equal to zero).**

110. Write down several ratios equal to a : b (using the previous property).

111. Multiply simultaneously both terms of the ratio $\frac{1}{3} : \frac{2}{5}$ by the mumber 15, which is the least common multiple of the denominators of the fractions $\frac{1}{3}$ and $\frac{2}{5}$. Will the terms of the ratio obtained be fractional numbers ?

112. Substitute a ratio of whole numbers for the ratio of fractional numbers given below :

(a) $\frac{3}{4} : \frac{1}{5}$ (b) $2 : \frac{1}{8}$
(c) $3\frac{1}{2} : 3$ (d) $1\frac{1}{3} : 2\frac{1}{4}$
(e) 2·13 : 0·5

113. Ratio of two numbers is equal to 20.
    (a) Find the consequent term of the ratio, if the previous term is 1·8.
    (b) Find the previous term of the ratio, if the consequent term is 2·5.

114. Solve the following equations :

(a) $3 : x = 1{\cdot}2$ (b) $2{\cdot}8 : x = 200$

(c) $x : 0{\cdot}03 = 100$ (d) $x : 1\frac{1}{3} = 12$

(e) $0{\cdot}15 : x = 1\frac{1}{3}$

115. The distance of 650 km between two towns pictured on a map is shown by a segment 13 cm long. Calculate the numerical scale of the map

116. (1) The numerical scale of a plan is $\frac{1}{200}$. How long will be the segment on the plan, if the distance on the ground is 20 m, 120 m ?

(2) The numerical scale of a plan is $\frac{1}{10000000}$. How long will be the segment on the map, if the corresponding segment on the ground is 100 km, 800 km, 3000 km ?

Let us consider two ratios, $2 : 5$ and $3 : 7\frac{1}{2}$. It is easy to be convinced after calculation that these ratios are equal. Therefore, it is possible to equate these ratios. We get $2 : 5 = 3 : 7\frac{1}{2}$.

Now we have obtained a proportion.

Definition : **A statement showing the equality of two ratios is called a proportion.**

117. Is it possible to write a proportion from the following ratios ?

(1) $43 : 14$ and $72 : 23$
(2) $78 : 13$ and $60 : 12$
(3) $3{\cdot}5 : 21$ and $2\frac{1}{4} : 13\frac{1}{2}$
(4) $0{\cdot}1 : 0{\cdot}02$ and $4 : 0{\cdot}8$

Make it a rule to read a proportion in the following way : ‘2 is to 5 is the same as 3 is to $7\frac{1}{2}$’. It is possible to read the same proportion the other way, namely, ‘5 is as many times of 2 as $7\frac{1}{2}$ is of 3’. Or ‘2 is the same part of 5 as 3 of $7\frac{1}{2}$’. The numbers, completing a proportion, are called the terms of the proportion. A proportion consists of four terms : For the proportion $2 : 5 = 3 : 7\frac{1}{2}$, 2 and $7\frac{1}{2}$ are called the *extreme terms* of the proportion, as they stand at the edges of the proportion.

5 and 3 are called the *mean terms* of the proportion, as they stand in the middle of the proportion.

118. Check the following proportions :

(1) $4 : 14 = 14 : 49$
(2) $10{\cdot}2 : 0{\cdot}65 = 0{\cdot}17 : 0{\cdot}055$
(3) $25 : 0{\cdot}5 = 1250 : 25$
(4) $24 : 3 = 36 : 4$
(5) $4\frac{1}{2} : 3\frac{1}{2} = 27 : 21$
(6) $5\frac{1}{3} : 8 = 3\frac{3}{7} : 5\frac{1}{7}$

Let us consider the proportion :

$6 : 3 = 8 : 4$

Multiply separately the extreme and mean terms of the proportion.

Product of the extreme terms is equal to $6 \times 4 = 24$ and the product of the mean terms is equal to $3 \times 8 = 24$. The product of the extreme terms of a proportion is equal to the product of the mean terms of the proportion. Let us show that it is true of any proportion $a : b = c : d$.

Let us multiply both parts of that equality by the same number $(b \times d)$

$(a : b) \times b \times d = (c : d) \times b \times d$

$(a \times b \times d) : b = (c \times b \times d) : d$

As a result we obtain

$ad = bc$

Conclusion : **The product of the extreme terms of any proportion is equal to the product of the mean terms of the same proportion.**

The above property of a proportion is called the basic property of a proportion.

119. What is the basic property of the proportion $a : 3 = b : 1$?

120. Write down a proportion corresponding to each equality given in the form of the basic property of a proportion.

(a) $1 \times 6 = 2 \times 3$ (b) $2 \times 15 = 6 \times 5$

(c) $2{\cdot}5 \times 0{\cdot}018 = 0{\cdot}15 \times 0{\cdot}3$

(d) $2\frac{1}{2} \times 1\frac{2}{7} = \frac{5}{7} \times 4\frac{1}{2}$

121. Write down the corresponding proportions from the following equalities :

(1) $15 \times 42 = 35 \times 18$

(2) $54 \times 55 = 66 \times 45$

(3) $7{\cdot}5 \times 12{\cdot}6 = 3\frac{1}{2} \times 27$

122. Is it possible to complete a proportion from the following numbers ?

15, 3, 4 and 20

123. Complete different proportions from the following numbers :

(1) 0·16, 0·32, 0·4 and 0·8

(2) $\frac{2}{3}$, $\frac{5}{6}$, $\frac{3}{8}$ and $\frac{3}{10}$

(3) $3\frac{1}{3}$, $5\frac{1}{4}$, $4\frac{1}{2}$ and $3\frac{8}{9}$

(4) 1, 2, 4, 8, 16 and 32

124. Three numbers 2, $\frac{1}{2}$ and 5 are given. Find the fourth number so that it would be possible to complete a proportion from the four numbers. How many solutions can this problem have ?

The basic property of a proportion gives us the clue to calculate any unknown term of a proportion. Let us consider the proportion.

$$70 : 10 = 21 : x$$

Let us write the basic property of this proportion :

$$70 \times x = 10 \times 21$$

$$\text{from this } x = \frac{10 \times 21}{70} \text{ or } x = 3$$

Conclusion : **Each extreme term of a proportion is equal to the product of the mean terms of the proportion divided by the other extreme term.**

125. Solve the following proportions :

(1) $x : 16 = 3 : 6$ (2) $x : 15 = 8 : 24$

(3) $108 : 90 = 42 : x$ (4) $72 : 40 = 324 : x$

(5) $x : 12 = 4\frac{3}{4} : 7\frac{1}{8}$ (6) $x : 1\frac{3}{7} = 1\frac{1}{5} : 1\frac{1}{3}$

(7) $15{\cdot}6 : 2{\cdot}88 = 2{\cdot}6 : x$ (8) $1{\cdot}25 : 1{\cdot}4 = 0{\cdot}75 : x$

How do you calculate the unknown mean term of a proportion ?

Let us consider the proportion $24 : x = 8 : 5$

Let us write the basic property of the proportion :

$24 \times 5 = 8 \times x$ From this $x = \frac{24 \times 5}{8}$ or $x = 15$

Conclusion : **Each mean term of a proportion is equal to the product of the extreme terms divided by the other mean term.**

126. Solve the following proportions :

(1) $36 : x = 54 : 3$ (2) $75 : 35 = x : 14$

(3) $343 : 98 = x : 60$ (4) $6\frac{1}{2} : x = 6\frac{5}{8} : 4{\cdot}1$

(5) $0{\cdot}38 : x = 4\frac{3}{4} : 1\frac{7}{8}$ (6) $3\frac{1}{2} : 0{\cdot}4 = x : 1\frac{1}{7}$

While solving the problems in a proportion, it would be often very convenient and easy if the places of terms of the proportion are interchanged.

What transpositions are possible to be made in a proportion ?

Let us consider the proportion $3 : 5 = 12 : 20$ and let us see whether, after the following transpositions, we get a proportion.

(1) Interchange the extreme terms of the proportion, we get the proportion $20 : 5 = 12 : 3$.

(2) Interchange the mean terms of the proportion, we get the proportion $3 : 12 = 5 : 20$.

(3) Interchange simultaneously the extreme and the mean terms of the proportion, we get the proportion $20 : 12 = 5 : 3$.

(4) Put the extreme terms in the place of the mean terms and put the mean terms in the place of the extreme terms. We get the proportion $5 : 3 = 20 : 12$.

Let us carry out the three previous transpositions in the proportion obtained in (4). We get :

(5) $12 : 3 = 20 : 5$ (6) $5 : 20 = 3 : 12$

(7) $12 : 20 = 3 : 5$

And so we obtained more proportions from the given proportion by interchanging the terms of the proportion.

127. Carry out all possible transpositions of the terms of the proportions :

(1) $12 : 2 = 20 : 5$ (2) $5 : 15 = 4 : 12$

(3) $a : b = c : d$

128. Simplify the proportions by making them free from fractional terms

(1) $\frac{1}{4} : \frac{3}{8} = 20 : 30$ (2) $\frac{1}{2} : \frac{1}{48} = 20 : \frac{5}{6}$

(3) $5\frac{1}{3} : 8 = 3\frac{3}{7} : 5\frac{1}{7}$

Proportional Values : Let us consider the following problem :

A person moving along a straight line with a uniform velocity, covers 12 cm in each second. Find the distance travelled by that person in 2,3,4. . . .10 sec.

Let us compute a table of the distance travelled in relation to the time.

time (sec.)	1	2	3	4......	10 .....
distance	12	24	36	48......	120.....

From the table we see that if we increase (decrease) the first variable (time) by a certain number of times, the second variable (distance) is increased (decreased) by the same number of times (*velocity is uniform*).

Two such values are called *proportional values.*

129. 15 m of cloth costs Rs. 60. Calculate the cost of the cloth whose lengths are given in the table.

length in metres	1	2	3	4	15	18	20
cost in rupees					60		

Would these values be proportional ?

If we denote the time or the length of the cloth in the above examples by the letter 'x' and the constant speed or price of one metre of cloth by the letter 'k' and the distance or cost of the cloth by the letter 'y' we get $y=kx$

The formula obtained above determines proportional dependence between the values of x and y.

130. In which of the following examples do you observe proportionality ?

(1) number of the rotations of the driving wheel of the boiler and its speed

(2) height of a man and his weight

(3) weight of one nail of the given size and its number in one kg

(4) distance travelled by rail and the fare

(5) number of rotations of a wheel for a given distance and the diameter of the same wheel

(6) number of telegraphic poles on a given road and the distance between the telegraphic poles

(7) time of burning of a lamp and quantity of fuel consumed

(8) speed and the distance covered in a given time

(9) speed of a moving body and time which is necessary for covering a given distance

(10) side of a square and its area

(11) length and width of a rectangle for a given area

(12) edge and volume of a cube

131. Complete the tables given below if it is given that X and Y are either proportional or inversely proportional :

1.

X	3	6	12	24	48
Y			8	16	

2.

X	3	6	12	24	48
Y		16	8		

3.

X	27	54	12	81	36
Y		6		4	

4.

X	24	54	12	81	36
Y	8				12

Having considered the table of proportional values (quantity of goods and price) or inversely proportional values (speed and time), we see that to each quantity of the first variable there corresponds one and only one quantity of the second variable.

In the case of proportional values, the ratio of two quantities of the first variable is equal to the ratio of the corresponding two quantities of the second variable.

For example, in table (1) ratio of two values of $X=12:24=\frac{1}{2}$ and the ratio of the corresponding values of $Y=8:16=\frac{1}{2}$.

But in table (4) ratio of two values of $X=24:36=\frac{2}{3}$ and the ratio of the corresponding values of $Y=8:12=\frac{2}{3}$.

In the case of inversely proportional values, the ratio of any two quantities of the first variable is equal to the inverse ratio of the corresponding quantities of the second variable, *i.e.*,

(1) ratio of two values of X in table (2) $=6:12=\frac{1}{2}$
and the ratio of the corresponding values of $Y=\frac{16}{8}=\frac{2}{1}$

(2) ratio of the two values of X in table (3) $=54:81=\frac{2}{3}$
and the ratio of the corresponding values of $Y=6:4=\frac{3}{2}$

These properties of proportional and inversely proportional values are used while solving different kinds of problems.

132. (1) 800 g of solution contains 50 g of salt. How much of salt is in 1 g of solution and in 240 g of solution ?

(2) In 1 kg of sugar there are approximately 125 cubic pieces. Find the average weight of one piece. How many pieces are there in 400 g of sugar ?

133. For making 800 copy-books, it is necessary to have 68·8 kg of paper. How much of paper is necessary for making 1200 copy-books ?

134. (1) The weight of 15 litres of petroleum is equal to 12·3 kg. What is the weight of 35 litres of petroleum ?

(2) We can extract 5·4 kg of oil from 24 kg of cotton seed. How much of cotton seed is necessary in order to extract 7·2 kg of oil ?

135. 100 cu m of air contains 21 cu m of oxygen. Determine the volume of oxygen in the room, the length, width and height of which are 10 m, 8 m and 3·25 m respectively.

136. (1) To construct a fence round the school ground, we require 76 poles if we fix them in such a way that the distance between the two consecutive poles is 4·5 m. How many poles will be required if we keep the distance between the two consecutive poles as 3·6 m ?

(2) A wheel with a circumference of 1·5 m makes 96 rotations to cover a certain distance. How many rotations should a wheel with a circumference 2·4 m make to cover the same distance ?

137. A school received 600 cypy-books of 20 sheets each and it was asked to remake them into copy-books of 12 sheets each. How many copy-books did the school get after remaking ?

138. If we consume 3·6 tonnes of coal per day, the coal will be sufficient for 45 days. For how many days will the coal be sufficient if we consume 2·4 tonnes per day ?

139. Rice contains 75% of starch and barley contains 60% of starch. How much of barley is necessary in order to extract the same amount of starch, which we can extract from 5 kg of rice ?

140. A watch was slow by 10 sec in 3 hours. How much slow would the watch get in 8 hours, 12 hours, 24 hours and a week ?

141. A kettle, filled with water at 18° C, takes 20·5 mts to boil. In how many mts will the water in this kettle boil under the same conditions if it is filled with water at 4°C ?

142. (1) Two pulleys are connected by an endless strap. The circumference of the first pulley is 28 cm and that of the second is 42 cm. How many rotations per minute will the second pulley make, if the first pulley makes 600 rotations per minute ?

(2) Two pulleys are connected by an endless strap. One of them makes 560 rotations per minute, and the other 240 rotations per minute. Find the circumference of the second pulley if the circumference of the first pulley is 0·36 m

143. The speed of a ship is to the speed of the current of water as 36 : 5. The ship went along the currrent for 5 hours and 10 minutes. How much time would it take to come back ?

144. From a cloth 88·5 m long and 1·05 wide, 25 coats can be made. How many coats can be made from a cloth 315 m long and 1·18 m wide ?

145. (1) In order to make 4 tents it is necessary to have 20 m of tarpaulin of 1·2 m width. How many metres of tarpaulin of 0·9 m width is necessary for making 6 similar tents ?

(2) A certain load can be shifted by 5 trucks, each of 1·5 tonnes capacity, in 6·4 hours. How much time will 3 trucks, each of 2 tonnes capacity, take to shift the same load ?

146. On an iron plate which is 32 mm thick, it is possible to make 24 holes in 54 minutes. How much time will it take to make 16 similar holes on an iron plate 28 mm thick ?

147. One universal loader (a big machine) can do in one hour what three loaders (smaller machines) can do in 8 hours. 18 loaders were found necessary to complete a certain work in 6 hours. In how much time would two universal loaders complete the same work ?

148. From a pond in the form of a rectangular parallelepiped, filled with water to a height of 1·2 dm, water was pumped

out in 16 hours by means of a pump. How much time would it take to pump out water of a height of 1·5 dm from the same pond by a pump, if the ratio of the productivity of the two pumps is 3 : 5 ?

7. Percentage

In the previous chapters you learnt about fractions and decimals. Some fractions are known by special names. For example, the fraction $\frac{1}{2}$ is called 'half', the fraction $\frac{1}{4}$ is called 'quarter' and the fraction $1\frac{1}{2}$ is called 'one and a half'.

Fractions which have 100 in their denominators form a special class.

The fraction $\frac{1}{100}$ or its equivalent 0·01 is given the special name 'one percent'.

Similarly 0·02 is called 'two percent' and 0·11 is called 'eleven percent'. The symbol '%' is generally used for the word 'percent.'

Thus we can write the following equalities :

0·01=1% 0·02=2% 0·11=11%

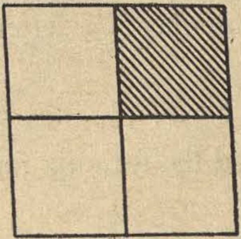

Fig. 4.13

149. Into how many equal parts should the square in fig. 4·13 be divided in order to find 1% of it ? What percentage of the original square is the shaded part ?

150. In the rectangular plot shown in fig. 4.14 the shaded part is occupied by cucumbers, and the rest of it by other

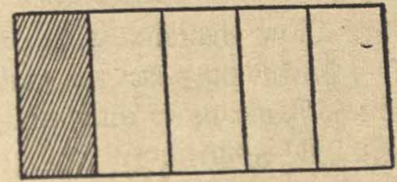

Fig. 4.14

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vegetables. What percent of the area is occupied by cucumbers ? What percentage of the area is occupied by the other vegetables ?

We can write a number in the form of a percent. To convert a given number into a percent, we have to find how many one hundredths does the given number contain ?

For example, 0·7 contains

(0·7÷0·01)=70 hundredths,

Therefore, 0·7=70%

Similarly, $\frac{2}{5}$=0·4=(0·4÷0·01)% or 40%, $\frac{2}{3}=(\frac{2}{3}\div 0{\cdot}01)$ % or $(\frac{2}{3}\div\frac{1}{100})$% or $66\frac{2}{3}$%

151. Write the following numbers in the form of percentages :
0·1, 0·35, $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{3}{4}$
$\frac{1}{8}$, $\frac{1}{50}$, 1, 0·042, 0·785, 3·05, $\frac{1}{6}$, $\frac{1}{25}$, 2, $3\frac{1}{2}$

152. Write the following percentages in the form of fractions :
5%, 20%, 72%, 100%, $\frac{3}{4}$%, 74%, 200%, 375%, $35\frac{5}{7}$%,
8%, 25%, $14\frac{1}{7}$%, 22·5%, 162·65%, $6\frac{1}{4}$%

Let us denote any number by letter a and its corresponding percentage by letter b.

Then, b=a÷0·01 or $b=a\div\frac{1}{100}$, *i.e.*, b=100a

Now we have obtained the formula for converting any number into the form of a percent.

We can use the formula $a=\frac{b}{100}$ for converting a percent into the form of a fraction.

Illustrative Examples

Problem 1. There are 40 pupils in a class, of which 45% are boys. How many boys are there in the class ?

Solution : First of all we shall find what part of all the pupils do the boys form ? For this purpose we shall write percentage in the form of a fraction : 45% means 45 one-hundredths or 45%$=\frac{45}{100}$.

Number of boys=45% of 40$=\frac{45}{100}\times 40=18$ (on simplification)

Therefore, there are 18 boys in the class.

Problem 2. There are 'a' fruit trees in a garden. P% of them are apple-trees. How many apple-trees are there in the garden ?

Solution : $P\% = \frac{P}{100}$ and $\frac{P}{100}$ of $a = \frac{a \times P}{100}$

If we denote the number of the apple-trees in the garden by letter b, than $b = \frac{a \times p}{100}$

We can obtain a formula for finding the percentage of any number.

153. Given a sequence, each term of which is equal to 60% of its previous term.
    (i) If a and b are the first two terms of this sequance, express b in terms of a.
    (ii) If the first term of the sequence is equal to 2500, find the first five terms of the sequence.

154. A sum of Rs. a was sanctioned for buying school equipment. 15% of this money was spent on buying a steel almirah. What amount was spent on buying the steel-almirah ?

155. An angle is given to be equal to 30% of a right angle. Find the value of the angle in degrees.

156. Compare 1·3% of 8·5 and 8·5% of 1·3.

157. Find :
    (i) 10% of 5 and 5% of 10
    (ii) 15% of 50 and 50% of 15
    (iii) a% of b and b % of a
    What conclusion do you draw ?

158. Prove that m% of the sum of two numbers a and b is equal to the sum of m% of a and m% of b.

To find a number if its several percentages are known.

Example 1. On a particular day a shopkeeper sold butter worth Rs. 375. If this sale be equal to 20% of his total sale on that day, find the total amount of sale on that day.

Solution : $20\% = \frac{20}{100}$ and so $\frac{20}{100}$ of the total amount of sale is Rs. 375.

The total amount of sale $= \frac{375 \times 100}{20} =$ Rs. 1875

Example 2. What is grown in 'a' ha, is P% of the whole field. What is the area of the whole field ?

Solution : 1. $P\% = \frac{P}{100}$

2. $\frac{P}{100}$ of the whole field is equal to 'a' ha. Therefore, area of the whole field will be $\frac{a \times 100}{P}$ ha

If we denote the area of the whole field by 'b' we shall obtain the formula $b = \frac{a \times 100}{P}$ or $b = \frac{a}{P} \times 100$

159. After two consecutive reductions of prices by 20% and 25%, the price of a book became 45p. What was the initial cost of the book ?

160. An album and a book together cost Rs. 2·30. The cost of the album is 15% of the cost of the book. What is the cost of the album and the cost of the book ?

161. A rectangle, 7·6 m long and 1·2 m wide, is divided into two parts so that the area of one part is less than the area of the other part by 10%. Determine the area of each part of the rectangle.

Example 3. Out of 40 pupils in the fifth class 18 are girls. What is the percentage of the girls in the class ?

Solution : First of all we find what part of the total number of its pupils is the number of girls. For this it is necessary to divide 18 by 40. The girls are $\frac{18}{40}$th part of the class.

$$\frac{18}{40} = \frac{18 \times 100}{40}\% = \frac{1800}{40}\% = 45\%$$

∴ The girls form 45% of the class

Example 4. Out of *a* pupils of the sixth class b pupils are girls. What is the percentage of the girls in the class ?

Solution : The number of girls in the class $=\frac{b}{a}=\frac{b\times100}{a}\%$

If we denote the percentage of girls in the class by P, we shall obtain the formula $P=\frac{b\times100}{a}\%$ or $P=\frac{b}{a}\times100\%$

162. Two kinds of tea were prepared. In the first, 30 g of sugar was mixed with 190g of tea. In the second, 40g of sugar was mixed with 270 g of tea. Which tea would be more sweet ?
163. What precent is 0·3a of a, 0·16ab of ab, 1·2 m of m and 2·5abc of abc ?
164. The cost of a pencil is 22 paise and the cost of a note-book is 25 paise. By what percent is the cost of the pencil less than that of the note-book? By what percent is the cost of the note-book greater than that of the pencil ?
165. The price of a book was first increased by 20% and later on it was reduced by 20%. What will be the cost of the book, if its cost prior to revision was 72 p, a p ?
166. What will be the percentage of increase in the area of a square if each of its sides is increased by 10% ? Make a drawing.
167. Two segments are given. The length of the first segment is 80% of the length of the second segment. By what percent should the length of the first segment be increased in order to make it equal to the second segment ?
168. The speed of a train is 40km/h and it takes 6 hours to travel from one station to another. If the speed of the train is increased by 15%, by what percent would the time of the journey be decreased ?
169. A tourist covered the distance between two places in three days. On the first day he covered 40% of the distance ; on the second day he covered 37·5% of the

distance and on the third day he covered the remaining 13·5 km. What is the distance between the two places ?

170. 17·1 kg of copper was extracted from 112·5 kg of copper ore. What is the percentage of copper in the copper ore ?

171. Take a number, increase it by 40% and multiply the result by 16. If from this product 37 is subtracted, we get 75. Find the number.

172. The length of a rectangle was increased by 10% and its breadth decreased by 10%. By what percent would the area of the rectangle change ?

173. A number was increased by 20%. By what percent should this number be decreased in order to obtain the original number ?

174. In a farm, gram, wheat and barley are grown in 75, 70 and 50 ha respectively. In the remaining 20 ha vegetables are grown. Find the percentage of the total land under the different crops.

175. Measure the length and breadth of your class-room and calculate its area. Measure the lengths and breadths of windows and ventilators and find their area. What percent of the area of the class-room is the ventilation area ?

176. Mark two points on a sheet of paper so that the distance between them would be approximately 10 cm. Measure this distance with the help of a ruler. What is the error made ? Express this error as a percentage ?

177. It is generally considered that the whole arm-spread of a man is approximately equal to his height. By what percent does your arm-spread differ from your height ?

178. A rope, 19·8 m long, is cut into two parts so that the first part is longer than the second by 20%. Find the length of each part.

179. A book has 160 pages. On the first day a girl read 7·5% of the book. On the second day she read 8 pages more. What percent of the pages is yet to be read ?

180. Two boys Ram and Gopal collected stamps. The total collection is 420 stamps ; Ram collected 10% more than Gopal. Later on Gopal received 50 stamps as a gift. Find by what percent are the stamps with Gopal more than the stamps with Ram ?

181. Before drying, the moisture in the grain was 23% and after drying it was reduced to 12%. By what percent was the weight of the grain reduced ?

8. Simple Interest

182. From the formula $I=\frac{P\times R\times T}{100}$, find the value of I when:

(a) $P=800$, $R=3$ and $T=4$

(b) $P=480$, $R=3\frac{1}{2}$ and $T=2$

Find expressions for P, R and T separately from the above formula.

183. Calculate P from the expression above when :

(a) $I=60$, $R=5$ and $T=n$ where $n=1, 2$, or 3. Can n be equal to 0 ?

(b) $I=60$, $T=4$ and $R=x$

Give x the values 1, 2, 3 and find P in each case. What happens if $x=0$

You know that when we buy things from the market, the shopkeeper charges some extra money along with the cost of the particular commodity, as his profit. Again when we get a house on rent or hire some crockery and tents for celebrating a function, we pay rent for the house, crockery and the tents used. Similarly, when we borrow some money or use some one's money for a particular need, we make some extra payment to the lender. This extra payment is called *Simple Interest*. Banks and Post offices also pay us interest when we deposit our money with them. Some times the government of a country collects loans on interest for a certain period of time for constructing dams and big projects. Simple interest is calculated with the help of the formula $I=\frac{P\times R\times T}{100}$ where

P is the *Principal* or money *borrowed or lent*, R stands for the *Rate* of interest and T represents the *Time* or the number of years for which the interest I is calculated.

184. Find the simple interest payable in the following cases :

(a) P=Rs. 250, R=4% and T=2 years

P=Rs. 1200, R=6% and T $1\frac{1}{2}$ years

P=Rs. 2000, R=$5\frac{1}{2}$% and T=6 month

185. Complete the following :

(a) T and R being same if P is doubled, I is..................

(b) P and R being same if T is doubled, I is..................

(c) P and T being same if R is halved, I is..................

186. Shankar borrowed Rs. 800 at the rate of $5\frac{3}{4}$% for his sister's marriage What interest would he pay if he clears the debt after x years ? If x=1, 2, and 3, calculate the interest paid.

187. I deposited Rs. 300 for 4 years at x% rate of interest in my savings account. What interest shall I earn from this deposit ?

Calculate the interest when x=$3\frac{1}{2}$, 4 and $4\frac{1}{2}$.

From the above two questions, it is clear that other things being same, simple interest varies, with the variation in time or rate. Interpret the result when time or rate is zero. What will happen if time or rate is negative ?

188. Ram had Rs. 840 in his savings account where the interest rate is 4%. What interest does he get after 6 months ?

189. Sham had a bank balance of Rs. 5400 for which he was getting simple interest at the rate of $4\frac{1}{2}$% per annum. He withdrew the money for constructing a house. What annual income does he lose because of this withdrawl ?

190. The half-yearly interest on a bond of Rs. 500 amounts to Rs. 12·50. Find the rate of interest calculated for this bond.

191. I borrowed Rs. 1000 from a friend for 6 months and paid him Rs. 20 as interest. At what rate was the interest calculated? Also, calculate the interest if the rate of interest be 1% less.

192. The interest charged by a banker on Rs. 600 at the rate of $7\frac{1}{2}$% per annum for x years is Rs. 105. Find x.

193. How long will it take for a certain principal to earn an amount of interest equal to itself, if the rate of interest is 5% per annum?

194. Mohan got an advance loan of Rs. 200 from his provident fund for buying a cycle to be paid back in 2 years' time. What amount has he to pay if the interest rate is 4% per annum.

(*Hint* : Amount=P+I)

195. Ravi deposited some money in the post office which doubled in 20 years at a simple rate of interest. In how many years will it treble itself? Also, find the rate of interest.

196. A radio dealer offers a radio set for Rs. 100 cash together with 20 easy instalments of Rs. 20 per month. Find the rate of interest charged if it was otherwise available for Rs. 450.

197. Dev wanted to sell his horse. Hem offered him Rs. 1000 in cash, while Sohan offered him Rs. 1025 payable after 6 months. If the money is worth 5% simple interest, which offer is better?

198. Hari paid Rs. 32 as interest for money borrowed for a period of 2 years. Had he paid the money after 3 years, he would have to pay Rs. 16 more. Find the money borrowed by him if the interest is 4% per annum.

199. Dyal borrowed some money from a money lender at simple interest. He paid Rs. 283·75 after a period of 3 years to clear off his debt. Had he paid after 4 years, he would have to pay Rs. 295. Find the money borrowed and the rate of interest charged.

Answers

CHAPTER 1

Page 2.

1. (a) 7, 3, 5 ; 4, 8, 6 ; 0, 6, 7 ; 5, 3 ; 9
2. (a) 222 (b) 244, 422, 424, 442, 224 and 242
   (c) 346, 364, 436, 463, 634, 643

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3. (a) 6 (b) 18 (c) 107
4. 12, 18, 21, 37, 9, 19
5. XIII, XVI, XVII, XXVIII, IV, IX, XXIX, XXXVI, XLI
6. I, II, III, IV, V, VI, VII, VIII
7. III means 'three' in Roman system and 111 in International system means one hundred and eleven.

Page 4.

8. No mistake
9. No
10. 8, 15, 190 and 8646
11. 357, 375, 537, 573, 735, 753

Page 6.

12. In 215 (5) in 5001 (5000) and in 2550 (500 for 5 which is 2nd from left and (50) for the next 5)
13. Any 4 numbers, *i.e.*, 4286, 4268, 4862, 4826
14. (a) 3, 8, 9 (b) 38, 83, 89, 98
15. 1234321. There can be other numbers also.
16. From right the place values in 5426025 are 5(5), 2(20), 0(0), 6(6000), 2(20000), 4(400000), 5(5000000)
18. (a) $<$ (b) $>$

Page 7.

19. 1
20. (a) $300000 > 3000$ (b) $5555 > 0$
21. (a) false (b) false (c) false (d) true

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22. 1, 14, 606, 874, 10001 ; $1 < 14 < 606 < 874 < 10001$
23. $400 > 210 > 86 > 45 > 5$
25. 215, 225

26 Red=4, green=2 and blue=1

27. $202 < 380$ and $380 > 202$; $202 < 445$ and $445 > 202$; $380 < 445$ and $445 > 380$
28. A=4, B=3, C=1 ; A=5, B=2, C=1

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29. 4 **30.** yes **31.** (a) 1000099 (b) 10000099

32. (a) 5651 (b) 4207

Page 10.

37. 50 + 52 ; 101 + 103

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44. (a) 1000, (b) 1200

45.

21	16	13	12
22	17	14	15
23	18	19	20
24	25	26	27

46. (a) 48730 (b) 3351

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48. yes **49.** 1 **50.** 1089

51. Many answers like 2806, 5 and 2816, 15

52. Many pairs like 782, 3 and 784, 5 **53.** 900000001

54. Pairs like 3003, 9 ; and 893, 2119 ; ....

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56. (a) 800 (b) 1200 **57.** (a) first > second (b) second > first

58. (a) 57 + 75 (b) 86 − 64

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60. Increasing at the rate of 25 t/h

61. 52 km/h **62.** 9 km/h

63. 37t − 18t + 12t or 37t + 12t − 18t

64. (a) 1075 (b) 1808 (c) 48000 (d) 2505

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68. (a) first > second (b) first > second

69. (1) $x+14=25$ (2) $42-x=18$ (3) $x-15=52$

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70. (a) 15 (b) 32 (c) 75 (d) 12

71. (a) $x=18$ (b) 32 (c) 2 (d) 5

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72. (a) The sum of two numbers is 70. If one of them is 32, find the other. Or what should be added to 32 to make it 70 ?
(b) The difference of two numbers is 46. If greater of them is 96, find the smaller. Or what should be subtracted from 96 to get 46 ?
Similarly for parts (c) and (d) **75.** $x=45$

76. (a) $x=1995$ (b) $a=252$ (c) $a=416$ (d) $x=1828$ (e) $a=2379$ (f) $x=1889$

Page 19.

77. (a) 27 m (b) 38 mangoes
78. $x+185=320$ $\therefore$ $x=135$ m
80. (a) 25 (b) 10 (c) 5 (d) 0
81. (a) $x=53000$ (b) $a=58357669$

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82. (a) 18 p (b) 28 p
83. $x+28=82$ $\therefore$ $x=54$ mm **84.** (a) 209 (b) 89
85. (a) 0 (b) 8 or 9 (c) 9 (d) any one from 21, 22………29
86. no **87.** Hindi 3, Tamil 2, Urdu 1
88. when $x=0$
89. yes, when two numbers end in zero
90. yes, when subtrahend is zero. **91.** 14

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92. 34 yrs, 29 yrs. **93.** 123, 132, 213, 231, 312, 321
94. 2, 10, 18, 26, 34………..; 74
95. (a) 5 (b) when minuend and subtrahend have the same digit at unit's place. (c) when minuend and subtrahend have the same digits at tens' and units' place.
96. yes **97.** 55 yrs, 19 yrs **98.** 999900

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100. (a) yes, when one number is 1 (b) yes, when both of them are 1
101. (a) yes (b) yes **103.** 2881 paise or Rs. 28 and 81 paise

Page 24.

112. 355914 **113.** (a) 111540 (b) 727 (c) 287
115. (a) 1 (b) 2 (c) 0 (d) 0 (e) 1 (f) 1
116. (a) 0 (b) 4 **118.** 25 and 400 only **120.** no

Page 25.

121. 1. 3, 9, 27, 81, 243, 729, 2187…… ; 8th term

122. (a) $37=24+13$ (b) $58=82-24$
123. (a) $a=50+23$ (b) $600=5a$ (c) $220=a-30$
125. (a) $x+7=7357$ (b) $8x=496$
128. (a) 35640 (b) 0 (c) 0 (d) 25 **129.** (a) 5 (b) 6

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131. 4, 8, 12,……… **132.** (a) $38=18+20$ (b) $a=45-10$
133. $420x=8820$ (No. of days is x) **134.** 125000

Page 27.

135. (a) 31415 (b) 7500

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137. (a) $x=8$ (b) $x=3$ (c) $x=6$, $y=4$ (d) $x=2$, $y=3$
138. (a) 465 (b) 9090 (c) 6695
139. (a) 2955 (b) 9900 (c) 5960 **140.** 6400
142. (a) 792 (b) 472 (c) 5725

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143. (a) 2700 (b) 2400 (c) 6030 (d) 13000
144. (a) 28300 (b) 65000 (c) 9600 (d) 7900
145. (a) 66 cm (b) 120 cm
146. (a) 308000 (b) 2121000 (c) 154025 (d) 455975
147. (a) 920 (b) 3250 (c) 1440 (d) 770 (e) 1000
(f) 3000 (g) 2000 (h) 157

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149. (a) 202000 (b) 0 (c) 30300 (d) 300
150. (a) 19000 (b) 30000
151. (a) $(48+56)\times 2$ cm (b) $(400+a)\times 2$cm
152. (a) 364812 (b) 305915
153. (a) 3600 and 441 (b) 2800 and 12

Page 31.

154. 6, 12 and 8 **155.** BA, BC, BN ; no **158.** 88 cm

Page 32.

159. 5000 cm^2

Page 33.

162. yes, no **163.** 8 **164.** 12
165. (a) 16 (b) 220 (c) $40x$ **166.** 880

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167. (a) 600 (b) 16 a **168.** 27 cu cm **169.** 120 cu cm
170. 20 cm^2, 120 cm^3

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171. Many values of l, b and h like 24 cm, 1 cm, 1 cm ; 6 cm, 2 cm, 2 cm and 4 cm, 3 cm, 2 cm
172. (a) 24000 cm^3 (b) 18000 cm^3 (c) 19200 cm^3 (d) 8000 cm^3
173. (a) 27 cm^3 (b) 125 cm^3 (c) 27000 cm^3

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175. area of bottom=area of top=460 cm^2 ; area of each side face =276 cm^2 and area of each end face=240 cm^2
178. 5 elephants, 5 times

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179. (a) divisor (b) divisor (c) divisor **181.** 308 cm^3
182. 8 times

Page 38.

185. no

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186. (a) 1607 (b) 408 **188.** (a) 47 (b) 703
189. (a) 608 (b) 2571375 **190.** 8050×302
191. no **192.** 4, 16, 22, 908, 24330
193. by adding 2 to each preceding term ; 19 ; no
194. 3, 6, 9, 12, 15,…………; 75
195. (a) sum, difference and product all even
(b) sum and difference even but product odd
(c) sum and difference odd but product even

Page 40.

196. (a) $500 \div 100$ (b) $0 \div 335$ **198.** 49 days
199. (a) 42 (b) 516
200. 2048, 1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1 ; 413 is not in this row. Yes, 1 is the end of this row of natural numbers.
201. 10, 20, 30, 40,……… ; rule : 10 is added to each preceding number.
202. n, 2n, 3n………rule : multiply the row of natural numbers by n.
203. 7656 and 7704 **204.** 22
205. (a) $84=4 \times 21$, (b) $a=20 \times 8$, (c) $a=300-20$

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206. (a) $31+65 > 50$ (b) $80<336-97$ (c) $46\times 45 < 4000$
(d) $72 > 800\div 80$

208. (a) 445 (b) 901 (c) 241703 **209.** (a) 102 (b) 210 sq cm

210. (a) $x=102$ (b) 91

211. (a) $(20000-13030)\div 34$ (b) $163-(154\div 11)$ (c) $(4031+(154\div 11)$

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212. (a) $46\times 88>46+88$ (b) $0\times 68=0\div 68$ (c) $(408-1)<408\div 1$
(d) $30+8=380\div 10$

213. 2025 cm^3 **214.** (a) 177 (b) 410 (c) 40 (d) 72

215. 4 **216.** p **218.** (a) 3a (b) x (c) 1 (d) m

219. $x=24$ **220.** $x=7$

Page 43.

221. 60 **222.** table Rs. 36 and chair Rs. 4 **223.** 30 paise

224. $l=210$ m, $b=35$ m **225.** 20, 100 and 700 **226.** 6 times

227. 36, 39 **228.** 18, 9 **229.** 1201, 1202, 1203, 1204, 1205

230. 6081, 6083, 6085, 6087, 6089, 6091

232. (a) 420000 cu cm (b) 2200000 cu cm

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233. (a) 64000 cu dm (b) 1000 cu dm

234. (a) 84000 cu dm (b) 312000 cu dm

235. 1000000 cu m, 10000 sq cm, 100 cm **236.** 23800 sq cm

237. (a) 900 cu cm (b) 7800000 cu cm (c) 60000 cu dm

238. 30800, 0

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240. 22, 0 **241.** 49, 74, 99 ; yes **242.** In the first case

243. Saturday

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244. 15 **245.** (a) 16 (b) 943718

246. (a) yes (b) yes (c) not (d) not **247.** even

248. when two or four numbers are even, sum will be odd, otherwise even.

249. 28 **250.** 520 km, 0

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251. (a) 8810 (b) 16810 (c) 40 (d) 4410

252. (a) 7759 (b) 36 (c) 477 (d) 1262

253. 2 km, 500 m ; 7 km, 500 m

254.

0	1	3
8	0	5
7	6	0

There can be other solutions also.

255. (a) 0 (b) 7960 **256.** $5a+112$; 372 **257.** $4x+45$

258. (a) 27200 (b) 63000

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259. 82 **260.** x yrs, $x-33$ yrs, $3x-99$ yrs, no, 48 yrs, 15 yrs, 45 yrs

261. (a) 224 (b) 66

CHAPTER II

Page 51.

8. (a) +9, +17, +100 (b) −5, −37, −250 (c) 3, 47, 503 ; 0

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12. +15 means 15° above freezing point ; −3 means 3° below freezing point ; 0 means freezing point and + ; +44 means 44° above freezing point

13. (1) 3 km south (2) Rs. 10 lost (3) 500 m down (4) Rs. 20 spent (5) 12° south latitude (6) 4° fall in temperature

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21. (a) 239, 200, 5, 0, −3, −240……… (b) −6, −7, −8………

22. +129, +131 ; +799, +801 ; +209, +211 ; −301, −299 ; −1, +1

24. +1, −1 ; +2, −2 ; +3, −3 ;…….

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26. Nainital **27.** (a) $a < b$ (b) $a > b$

28. (a) $-240<+3$ (b) $96>-80$

29. $-40<-13<-7<0<+1<+7<+9$

30. $+29>+5>+1>0>-2>-25>-30$

31. (a) $a>0$ (b) $b>0$ (c) $0>m$ (d) $n<0$ (e) $a>m$
(f) m and n cannot be compared unless their magnitudes are known
(g) $m<b$ (h) $b>n$ (k) same as (f)

32. (a) positive (b) negative **33.** (a) negative (b) positive

34. (a) +24 (b) −36 (c) −2, −1 (d) 0 (e) −1, 0, +1 (f) −2

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35. (a) no (b) yes (c) no (d) yes
36. (a) $a<c$ (b) $a>c$ (c) $c>a$ (d) $c<a$
38. (a) $-45, -44, -43, -42$ (b) $-4, -3, -2, -1, 0, +1$
39. (a) -10 and 0 (b) 0 and $+8$ (c) -4 and $+2$
40. (a) $a<m$ (b) $a>m$ **41.** $+9>-5>-7$ or $-7<-5<+9$
42. (a) 70° mark (b) 20° mark

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46. (a) -30 (b) -80 (c) -34 (d) $+500$ (e) ${}^{+}20$ (f) $+900$
47. (a) $+16+{}^{-}6>-16$ (b) $-20+-4<-20$
(c) $-8+-5<-5$ (d) $-10+{}^{+}1<-1$
48. (a) false (b) false **49.** (a) yes (b) no (c) yes (d) yes
50. (a) -10 (b) -7

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53. (a) 0 (b) 0
54. $-17, +17$; $-8, +8$; $-6, +6$; $-4, +4$; $-3, +3$
55. $+200, +100, +65, +48, -84, -95, 0, +1$
56. Combinations like $+8+(+2)$; $-4+(-3)$; $+8+(+7)$; $-16+(-5)$
57. $+2+(-1)$, ${}^{+}1+({}^{-}2)$, $+3+(-1)$, ${}^{+}6+({}^{-}20)$. There can be many other pairs also.
58. (a) $x=+25$ (b) $a=-73$ **60.** $a+b=b+a$

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62. (a) -550 (b) $+10$ (c) $+16$ (d) -20
63. $11+a$ yrs ; 29, 14, 9, 11, 4, 16 yrs **64.** $+44°$
65. (a) $a+5>b+2$ (b) $a+5>b+2$ (c) $a+5>b+2$ (d) $a+5>b+2$

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66. (a) $+11$ (b) -2 (c) -1 (d) -9
67. $m-35$ yrs, If $m=33$ or 30 father is older than uncle by 2 or 5 yrs ; no.

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68. $(17-t)°$, 12°, 17°, 15°, 10°
69. (a) $+8$ (b) $+26$ (c) -2 (d) $+18$ (e) 0 (f) $+16$
(g) $+25$ (h) 0
70. (a) $-3-(+12)<18$ (b) $-2<+3-(-8)$
71. (a) true (b) false (c) false (d) true
72. (a) -13 (b) -12 (c) 0 (d) $+1$ **73.** when subtrahend is zero.
74. when subtrahend is equal to minuend
75. (a) $x=+42$ (b) $a=-97$ (c) $a=-81$ (d) -48 **76.** (a) $<$ (b) $>$

77. (a) 0 (b) -13 (c) $+10$ (d) -8
78. $+8, -25, +1, -68, 0$; $-7, +15, -1, +23, +36, 0$

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79. (a) $+4, -6, +20$ (b) $+2, -4, +5$
80. (a) $+1$ (b) -3 (c) $+9$ (d) $+9$
81. (a) $+13$ (b) -12 (c) $+2$ (d) -4
82. (a) $-28+(+32)$ (b) $(-46)+(+36)$ (c) $-x+(-a)$ (d) $x+(-m)$
83. (a) $(+4)+(+5)+(-6)+(-2)$ (b) $(-7)+(+7)+(-8)+(+4)$
(c) $a+(+b)+(-c)+(-m)$ (d) $(-a)+b+(+c)+(-m)$
85. (i) (a) 0 (b) -13 (ii) (a) -16 (b) -8

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88. 13
89. (a) $a+b+c+d$ (b) $-x-m+a-b$ (c) $-a-m+a+d$
(d) $m+k-a-m$
90. (a) $-3+4+(7-5)$ (b) $-m+a+(-n-b)$
91. (a) $-6-(2-4+3)$ (b) $a-(b+c+d)$ (c) $-a-(-b-c+d)$
(d) $m-n-(k+a-x)$
92. (a) $-10, -6$ (b) $-5, -13$
93. (a) $8-25+60$ (b) $-1+2-4$ (c) $-a-b+m$ (d) $a+b-c$
94. (a) 12 (b) -15

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96. (a) $-(6-8+7)+3-4$ (b) $-(-a-b+c)+m$ **97.** (a) -4 (b) $+13$
98. (a) -20 (b) -72 (c) -1200 (d) -168
99. -58 (b) $+4$

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100. (a) 30 (b) 160 (c) 100 (d) 375
101. (a) 16 (b) -53
102. $(-4)\times(-2)$; $(11)\times(-2)$; $(-41)\times(1)$; $(-8)\times(-4)$; $(-7)\times(-1)$
There can be other pairs as well.
105. (a) 360 (b) 0 **106.** $abc>0$ (b) $abc<0$ (c) $abc<0$ (d) $abc=0$
107. (a) negative (b) positive (c) positive (d) negative

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108. (a) -1470000 (b) -720
109. (a) $-4a$ (b) $-nb$ (c) $-8x$ (d) $-20a$
110. $(-a)\times 3$; $(-a)\times b$; $(-a)\times 1$
112. (a) $5a-5b+5c$ (b) $-6ab+6m-6n$ (c) $-2ab-4ac+6am$
(d) $-8am+12bm-20cm$

113. (a) $a(m-n-b)$ (b) $3(-m+a-b)$ (c) $ta(m-n+p)$
(d) $a(-bc+bm-cm)$ (e) $-4m(3a+2b+4c)$ (f) $-6x(4a+3b+5c)$

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114. (a) $b(a-b-c)$ (b) $-x(bc+ab+cd)$ (c) $ab(m+n+n)$
(d) $x(k-5-a)$ (e) $-5a(2b+3c+4b)$ (f) $a(36-10-13)$
115. (a) $-26n$ (b) $11kx$ (c) $-10nx$
116. $(-1)\times(40)$; $(-1)\times(-16)$; $(-1)\times(54)$; $(-1)\times(68)$; $(-1)\times(0)$
117. (a) -3000 (b) -24 (c) -720
118. (a) $ab > 0$ (b) $ab > 0$
119. (a) $x > 20$ when x is greater than 20 and $x < 20$ when x lies between 1 and 19 (b) $x < 20$ (c) $x < 20$
120. (a) $24a-16b+64c$ (b) $5am+4mx+6bm$
(c) $-12am+24an+6ak$ (d) $-15a+20m+40$

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121. (a) $19b$ (b) $-25mx$ **122.** (a) -4 (b) -3 (c) 3 (d) 0
123. (a) positive (b) negative

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128. (a) correct (b) wrong (c) wrong (d) correct (e) correct
(f) correct (g) correct (h) correct

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133. (a) true (b) false (c) true (d) false

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134. (i) $x=8$ (ii) $a=-3$ (iii) $x=10$ (iv) $m=2$ (v) $m=-5$
(vi) $x=-5$ (vii) $a=4$ (viii) $x=4$

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135. 230 kg **136.** 60 km **137.** 4 litres
138. hens=7, rabbits=12 **139.** Ram's step=63 cm, Mohan's step=51 cm
140. (i) $x=5$ (ii) $x=-5$ (iii) $x=1$ (iv) $x=1$
141. 602, 86 **142.** 914, 305 **143.** AB=19 cm, AC=41 cm, BC=34 cm

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144. 150 **145.** yes, no **146.** no, no **147.** 867 sq units
148. 300 km **149.** larger=84, smaller=16 **150.** 13 yrs
151. 3, 1, −1, −3,......; 722 is not in this row ; there is no end of this row.
152. Add −5 to the preceding term ; −20 ; −50
153. multiply the preceding term by −2 **155.** 1010

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156. 0 ; yes, the value 40 is extra. The value of the expression can be found when it is given that $a=c$,

157. 75 cm **158.** greater$=151$, smaller$=88$

159. 11 yrs **160.** $-1, -4, -7, -10$........; yes

161. row consists of even numbers with positive and negative signs alternately; only 38 is in the row ; -8 ; -28.

162. yes, 2nd and 22nd terms have the same number but with opposite signs.

CHAPTER III

Page 76.

2. $\frac{1}{6}, \frac{2}{6}, \frac{1}{8}$ **3.** $\frac{8}{25}, \frac{25}{40}$

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4. $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$ **5.** $\frac{1}{10}, \frac{1}{1000}, \frac{1}{1000000}$

6. $\frac{1}{100}, \frac{1}{10000}$ **7.** $\frac{1}{60}, \frac{1}{3600}$ **8.** $\frac{1}{2}$

9. 2 **10.** 4 **11.** 10

13. yes, by joining the diagonals

16. $\frac{1}{24}, \frac{1}{30}$ **17.** (a) 3 (b) 5

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20. (a) $\frac{1}{2}, \frac{4}{9}, \frac{3}{5}$ (b) $\frac{1}{2}, \frac{5}{9}, \frac{2}{5}$

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21. (a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ **22.** $\frac{4}{6}, \frac{3}{6}, \frac{7}{6}$

24. (a) 8 cm (b) 16 cm **25.** $\frac{4}{6}, \frac{5}{6}$

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26. Numerator : 8, 8, 7, 1, 25, 3
Denominator : 2, 12, 1, 8, 4, 40

28. Train leaves atter three fourths of an hour, train left half an hour before, train left three quarters of an hour before, train leaves immediately.

30. $\frac{53}{10}$ dm, $\frac{49}{10}$ dm (b) $\frac{71}{5}$ m, $\frac{1}{2}$ m

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33. (a) 89 mm (b) 263 mm
35. (a) 807·4 m and 0·08 m (b) 9·2 cm and 0·9 cm
36. (a) 214 cm, 8 cm (b) 240 p, 3 p

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37. 98 thousand four hundred and three point two seven six five.
39. (i) (a) 7·55 m, 2·49 m (ii) (a) 68·75 dm (b) 0·91 dm
(iii) (a) Rs. 2·29 (b) Rs. 0·03 (c) Rs. 0·12 (iv) (a) 5·008 t
(b) 0·009t (v) (a) 1·075 km (b) 3·009 km
40. (a) 25·07 (b) 55·70 (c) ·064 (d) 88·005

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41. (a) 27·40 m (b) 65·90 m
42. (i) (a) 4·750t (b) ·400t (ii) (a) 7q 80 kg (b) 5q 40 kg

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47. (a) 0 and 1 (b) 2 and 3 (c) −1 and −2 (d) −3 and −4
(e) 1 and 2 (f) 3 and 4 (g) 0 and 1 (h) −2 and −3
(f) 0 and 1
48. (a) −1·7 and −1·8 (b) −2 and −2·1 (c) −2·2 and −2·1
(d) −1·8 and −1·9
49. (a) > (b) < (c) < (d) <
50. −8·25, −0·3, 0, 0·13, 1·25, 5·621

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51. 14·1, 6·125, 0, −5, −7·3, −7·6
52. (a) 3, 4 (b) −6, −7 (c) −9 (d) −1, 0, 1
53. 9 **54.** Any digit from 3,4,5,6,7,8,9
56. (a) < (b) > (c) < (d) <
57. (a) −10, −9 (b) 0, 1
58. −4·5 > −8·52, 0·42 > −4·5, −8·52 < −4·5, −4·5 < 0·42, −8·52 < 0·42

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59. 25·3 m **60.** Rs. 1·78 **61.** 5·3 km
62. (a) 1253·6727 (b) 3·64 (c) 96·381 (d) 0·001
63. 7+·365 ; 8+·7 ; 0+·45

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64. 7−·41, 28−·2, 1−·619 **65.** 289 cm
66. (a) 0·232 (b) 30·933

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68. (a) 3·6+4·8 (b) −9·5+8·1 (c) 5·1+2·3 (d) −m+k
69. (a) 6·6, −2·4 (b) −0·9, −14·3 (c) −9·0, −1·8 (d) 10·2, −4·6
70. (i) (a) −1·0 (b) 1·7 (ii) −10·25
71. (a) −10 (b) 73·7 **72.** −42·87+36
73. 7·7−4·9 or 10·5−7·7

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74. (a) x=−6·52 (b) a=16·2 (c) a=−4·4 (d) m=15·5
(e) m=6·24 (f) x=−9·3
75. 4·7 **76.** 3·5 **78.** 16·7 km/h against, 26·5 km/h down
79. 21·2 km/h in still water, 25·9 km/h down
80. 4 km/h against, 9 km/h down **81.** Rs. 1·08
82. (a) 7·06 (b) 109·2 (c) 2·3 **83.** 63·3t

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84. 1·1 km south

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85. 0·25, 4·75=5 **86.** 3089=3000 **87.** 50 sq cm, 750=800
88. 30=31=28, 5 days **89.** 61 **90.** 9, 54, 98 or 99

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91. 9. 72, 5
92. (a) 67·3 (b) 4·9 (c) 305·0 (d) 0·9 (e) 85·5
(f) 902·2 (g) 0·4 (h) 99·8 (i) 859·5
93. 7, 46, 77, 102, 1, 0 **94.** (a) 0·8 (b) 1·5 (c) 2·5 (d) 52·7

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95. 0·07, 0·04, 10·08, 4·46 **96.** 2·314, 0·042, 30·005, 10
97. 90, 210, 5260, 2960 **98.** 8000, 30000, 306000, 279000
99. 76·5, 86·6, 7·7, 0·0, 1·0 **100.** 7·667, 0·222, 6·556
101. 7700, 700, 1000, 67600
102. 8000000, 870000000, 308987000000, 76201299000000

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103. both are equal
106. (a) Rs. 22·00 (b) Rs. 12·10 (c) Rs. 7·15 (d) Rs. 1·65

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107. (a) 38·22 g (b) 3·9 g **108.** 2·16 sq m
109. (i) 387·068 (ii) 56·9036 (iii) 0·536 (iv) 0·072 (v) 327·240
(vi) 10·64000

110. 386 dm^2 (b) 60 dm^2 **111.** (a) 7·820 (b) 0·552

112. (i) 0·0450 (ii) 0·167440 (iii) 270·9903 (iv) 6430 (v) 0·81
(vi) 0·060

113. (a) 803·7, $803·7>84·6$ (b) 88·83, $88·83>84·6$ (c) 84·6, $84\ 6=84·6$
(d) 4·33, $4·23<84·6$

114. (a) −0·48 (b) +0·30 (c) −1·47 (d) +26·5 (e) +3·43 (f) −1869

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115. 4·755, −64·95, 75 **116.** (a) −·337 (b) −7·920 (c) 1·4571
(d) −11·70

117. (i) −·9 ab (ii) −29·6 mn (iii) −2·95 ab (iv) −19·3 a
(v) −9m (vi) 5·9 n

118. −48·05, 5·58, −6200 **119.** (a) 29·52764 (b) 742·3744

120. (a) −333 (b) +4·567 **121.** 1·35

122. (i) $a=20$ (ii) $x=-1$ (iii) $x=+1$ or -1 (iv) $z=0$
(v) $p=-1$ (vi) $k=10$

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123. Rs. 2·6 **124.** 3m, 22cm **125.** 10·5 g

126. 3·60 cm **127.** (a) 0·125 (b) ·00823 (c) ·0125

128. (a) 3·24 (b) ·726 (c) ·856 (d) ·24596 (e) ·0003 (f) ·005

129. (a) 12·862 (b) 2·08 (c) 1·002 (d) 2·078 (e) 20 (f) 4

130. (a) $k=-2·05$ (b) $p=-6·06$ (c) $d=4·03$ (d) $m=-83·230$

131. 4·6 km/h

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132. 22·5 dm^3 **133.** 8·2 cm **134.** 26·4 dm^2

135. (a) 98·5 (b) 1·25 (c) 20·6 (d) ·0329

136. (i) $k=250$ (ii) $x=·125$ (iii) $c=·5536$ (iv) $p=50·5$

137. (a) 4625·4208 (b) 64·089 (c) 142·6 (d) 52

138. 4·5 times **139.** difference, 6·8 times **140.** 29·64, 24·44

141. 29·4, 21·0 **142.** 2, 5, 8, 11......... ; 14th term=41, yes

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144. 373, 375, 377, 379 **145.** 16·4 km/h, 2·2 km/h

146. cotton=48 p/m, linen=Rs. 6/m **147.** 36·75 kg, 14·7 kg, 26·25 kg

148. Rs. 24·20, Rs. 60·50, Rs. 169·40 **149.** 11·8 kg

150. 4 hrs 12 mts **151.** 5 km/mt **152.** 85 km/h

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153. $9m^3/h$ **154.** 55 km/h

CHAPTER IV

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1. $p=2\,(a+b)$; rule : perimeter of a rectangle is equal to twice the sum of its length and width.

3. $\angle ABD=180^\circ-a^\circ$, rule : one adjacent angle is equal to the difference of 180° and the other angle

4. $b=a+3$ **5.** multiply the previous term by -2 ; $-4, 8, -16, 32, -64$

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6. 340·48 kg ; 282·88 kg **7.** (a) 204 sq m (b) 8 sq m (c) 3·36 sq m

8. 5 cm, 2 cm

9. (i) -24° (ii) -18° (iii) -8° (iv) 0° (v) 10° (vi) 24°

F	-12°	0°	17°	32°	50°	76°	
C	-24°	-18°	-8°	0°	10°	24°	

Correct to 1°

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11. 120

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12. (a) $-4{\cdot}386$ (b) $0-{\cdot}136$ **13.** (a) true (b) true (c) true

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14. a mile $= a\times 1{\cdot}6$ km **15.** $a=10n$, 80 p

16. $b=3a$, 11th term $=-5$, 12th term $=-45$, 14th term $=-135$

17.

age in yrs	1	2	3
hrs. of sleep	$16\frac{1}{2}$	16	$15\frac{1}{2}$

18. 331m/sec, 341·8m/sec, 350·2 m/sec **19.** (a) 500·9 cm (b) 80·144 cm

20. 24

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21. 6·75 cm, 2·25 cm

22. (a) ·346 (b) 1·06 (c) $-15{\cdot}7438$ (d) $-5{\cdot}04$ (e) 1·068
(f) 9·563 approx. (g) 61·34 (approximately)

23. (a) 1 (b) 11·3175 (c) 26·26

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24. $\frac{a+b}{2}$, 14 **25.** $-6{\cdot}5$ **26.** $m=\frac{a+b+c+d}{4}$

27. $y=\frac{a+b+c+d+\ldots\ldots}{n}$ **28.** all are correct **29.** 4·65 cm, 1·55 cm

30. $S=\frac{ab}{100}$ ares

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32. a+b is the total weight of biscuits and sweet-meat, a−b is the difference in their weights (1) m=49·5, n=25·5 (2) m=325, n=85

33. a−1 **34.** 24 hrs, 26 hrs 10 mts

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35. d=s−5 (a+b), 70 km

37. 25 mts, both will reach the school at the same time.

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38. −0·75 **39.** (a) 2·526 (b) $1310\frac{6}{7}$ (e) 13·8875

40. s=100 ab **41.** p=a+2b **42.** 16 cm, 12 cm, 18 cm

43. a+b km/h ; a−b km/h ; ship will stand still in water ; stream will sweep away the ship with velocity b−a km/h **44.** $\frac{8b}{63}$ km/h

45. 54 km/h, 36 km/h **46.** 6 P.M.

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47. $I=40,\ P=\frac{I\times 100}{r\times t},\ r=\frac{I\times 100}{P\times t},\ t=\frac{I\times 100}{P\times r}$

49. $c=\frac{a+b}{a},\ a=2c-b$ **50.** $s=vt,\ v=\frac{s}{t},\ t=\frac{s}{v}$

51. a=bc, 0, by multiplying the quotient and the divisor

52. $a=bc+r,\ c=\frac{a-r}{b},\ b=\frac{a-r}{c}$ **53.** $t=\frac{c-b}{ab}$

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55. 0·032, no

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57. (a) −2t−2 (b) 5x−1 (c) 6y−19

58.

t	−3·4	−1·8	0·2
A	−15	−7	3

59. $x=-\frac{0{\cdot}9}{a}$ **60.** −26·5012

61. $k=xy,\ x=\frac{k}{y}$ **62.** $0,\ t=\frac{s}{v+w}$

63. (a) 5 (b) b (c) 12 a (d) 3

64. (a) $316=21\times15+1$ (b) $15=26\times0+15$ (c) $7236=36\times21+0$

65. $\frac{(2x+30)\times3-90}{6}$ **66.** 0·2, no

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67. $-9t+17$ **68.** $s=a^2$, 1·44 cm^2 **69.** $V=a^3$, 64 cm^3

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70. 226981, −243, 0·56733696, 128, 7·414875, 335·54432

71. 16·0 and 335·54432 ; −15 and −243

71.

Nos	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Sq. of Nos	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225	256	289	324	361	400

73.

Nos	1	2	3	4	5	6	7	8	9	10
Cubes of Nos	1	8	27	64	125	216	343	512	729	1000

74. 10, 100, 1000, 10000, 100000, 1000000, number of zeros is equal to the index.

75. ·1, ·01, ·001, ·0001, ·00001, ·000001, number of zeros between the decimal point and 1 is one less than the index.

76. (a) $10^2 < 2^{10}$ (b) $2^4=4^2$ (c) $3^4>4^3$

77. no, it is only true when $a=b$ or when $a=4$, and $b=2$, 4

78. (a) 1, 1, 1, 1, any natural number to the power 1 gives 1

(b) 0, 0, 0, 0, any natural number to the power 0 gives 0.

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79. −ve, +ve, +ve, +ve, +ve, +ve, +ve, −ve, neither +ve nor −ve

80. (a) 0 (b) $(-2{\cdot}4)^4$ (c) 7^4 (d) $(-2)^4$ (e) 1^5 **81.** no

82. (a) yes (b) yes (c) no **84.** yes

85. ±2, ±3, ±4, ±5, ±6, ±7, ±8, ±9, ±10

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86. (a) −508 (b) −35 (c) −125 (d) −125

88. (a) 15×10^7km (b) 43×10^7 (c) 51×10^7km^2, 149×10^6km^2, 361×10^6km^2 (d) 72×10^5 (e) 27×10^{18}